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The size of combustion turbine plants as affected by certain design parameters

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THESIS

THE SIZE OF COMBUSTION TURBINE PLANTS AS AFFECTED BY CERTAIN DESIGN PARAMETERS

Shelley Elmer Rule

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by

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Cambridge 39, Massachusetts
May 16, 1952

Professor J. P. DenHartog
Chairman, Departmental Committee on Graduate Students
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor DenHartog:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "The Size of Combustion Turbine Plants as Affected by Certain Design Parameters".

Sincerely yours,

Shelley Elmer Rule

THE SIZE OF COMBUSTION TURBINE PLANTS AS AFFECTED BY CERTAIN DESIGN PARAMETERS

BY

Shelley Elmer Rule
Lieutenant Commander, U.S.N.

B.S. in M.E., Georgia School of Technology
(1939)

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MECHANICAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
(1952)

Signature of Author: ______________________________
Department of Mechanical Engineering

Certified by: ______________________________
Thesis Supervisor

____
Chairman, Departmental Committee on Graduate Students

A B S T R A C T

THE SIZE OF COMBUSTION PLANTS AS AFFECTED BY CERTAIN DESIGN PARAMETERS

by

Shelley Elmer Rule

Submitted for the degree of Master of Science in Mechanical Engineering, on 16 May 1952.

As a means toward determining the effect of various parameters on combustion turbine plant size and weight for a particular net power output, three have been chosen as independent variables in which the plant may be expressed. These three are: the flow coefficient, (C_x/U); the blade length ratio, (L/d); the mass rate of flow, w. The size and weight of compressor, turbine and combustion components of a plant are stated in terms of these three, and the effect of blade aspect ratio on weight and volume is also mentioned. A specific example is used to illustrate the fact that the shaft speed ratio between compressor and turbine is intimately related to the optimum values of flow coefficient and blade length ratio to be chosen, whereas the effect of pressure ratio on this choice is very slight.

Thesis Supervisor: Warren M. Rohsenow

Title: Assistant Professor of Mechanical Engineering

ACKNOWLEDGEMENTS

The foundation upon which the latter part of this thesis rests, and which has been condensed to form the entire first portion, is the development of L. W. Shallenberg, presented as a thesis in 1951. I wish to express appreciation to the author both for that work and for the encouragement and assistance subsequently extended.

I desire also to express appreciation for the assistance and direction of Professor Rohsenow, the supervisor of this thesis, whose ever-willing advice contributed in large measure to the result.

TABLE OF CONTENTS

I. INTRODUCTION

The design of combustion turbine plants for optimum economy of construction and operation on the one hand, and for optimum weight and size against a specified useful output on the other, has been accompanied by much study, both along empirical and analytical lines.

In order to treat the problem by the ordinary mathematics, create a picture which can be grasped, cite an example which appears concrete and yet hold to a treatment sufficiently general to be useful, a considerable number of simplifying assumptions must be made -- and yet a minimum of them.

In the design of a power plant for many purposes, the weight of the plant and the amount of space it occupies are of paramount importance, whereas in any case both of them bear a relation to the first cost of the plant and its accommodation. Factors which may be thought to have an obvious bearing on size and weight may diverge from that widely in their actual effect.

The treatment herewith, as a small part of an extensive program, has drawn generously on what has gone before in attempting to focus on the effect of three particular variables on the size of rotating machinery: one geometric, one kinematic, and one a scale factor. The examination of each of the first two is limited to a span of "good practice", with the implication that trends shown within that

span will continue sufficiently far beyond it to reach the region, from the standpoint of each variable individually, of physical absurdity. Hence, if it is felt that the delineation of these spans of good practice is unjustified, a later extension or contraction of them will not alter the general aspect of the conclusions.

Subsequent to the treatment of rotating machinery, a general discussion of combustion equipment leads to relations, empirical but rational, for the weight and volume of that part of the plant.

Despite the fact that numerical constants have been inserted wherever possible to facilitate illustration, the influence of individual factors such as pressure ratio and cascade geometry is more truly represented than is the quantitative result. Even when the expressions arrived at permit a more general application, the tacit physical plant is a stationary or heavy propulsion plant in the 1000-10,000 horsepower range, but one borrowing from the light weight features of other types.

Whereas the method developed is in principle applicable to a wider range of plant configurations, only the simple CBT thermodynamic cycle between pressure ratios of five and ten has been investigated. An appropriate extension is to the CICBTX cycle, shown schematically in Fig. 1, using pressure ratios up to about fifteen. This is suggested as a subject for future study.

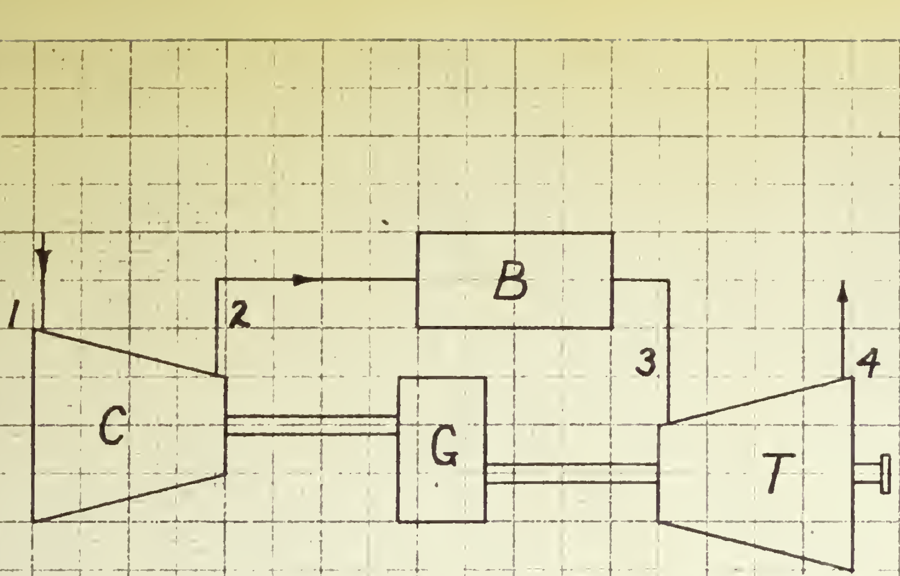
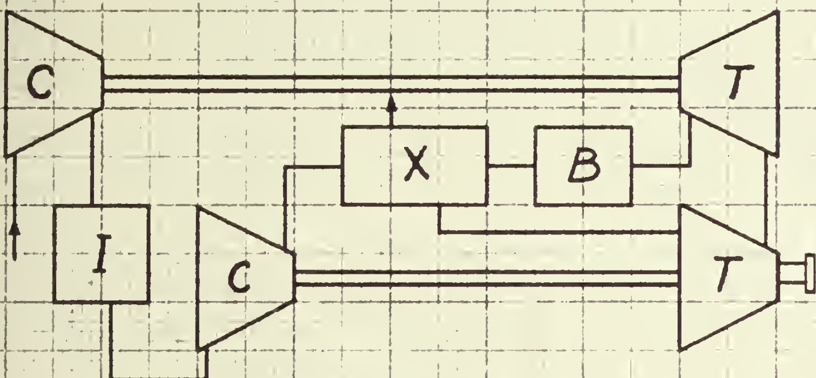


Fig. 1



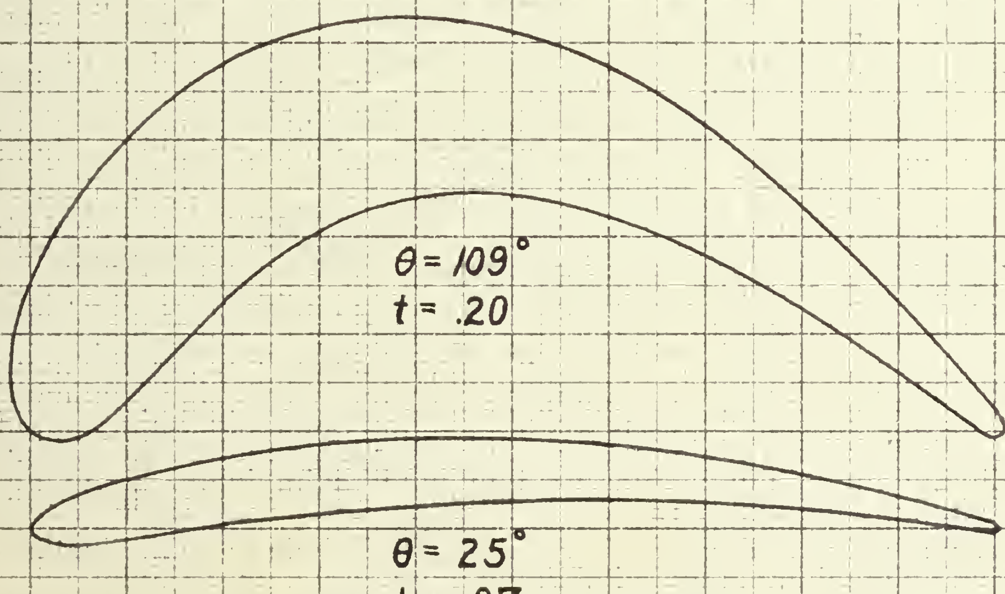


Fig. 3

II. BASIC RELATIONS AND DEFINITIONS

Assumptions involved in the analysis of plant components include: a constant pitch diameter throughout a particular machine, potential vortex flow, equal work done per turbine or compressor stage, a uniform axial velocity over the entire length of a machine. Compressor blading is assumed to be symmetrical in all stages except that the first and last are modified to meet substantially axial entry and discharge velocities. No pressure losses are considered in other than the turbine, and mechanical friction is overlooked. A mean value of specific heat is used in each of the ranges of compression, combustion and expansion, and the changes in mass flow and fluid properties due to fuel addition are neglected.

With these in mind, certain basic relations and some mathematical approximations which are used herein are outlined below.

Polytropic efficiencies:

$$\eta_c = \frac{r_c^{\nu} - 1}{(T_{02}/T_{01}) - 1} = \frac{r_c^{\nu} - 1}{r_c^{\lambda} - 1} = \frac{\eta_{sc}}{R_c^*}$$

$$T_{02} = r_c^{\lambda} T_{01} \tag{1}$$

$$\eta_t = \frac{1 - (T_{04}/T_{03})}{1 - r_t^{-\nu}} = \frac{1 - r_t^{-\mu}}{1 - r_t^{-\nu}} = R_t^* \; \eta_{st}$$

$$T_{04} = r_t^{-\mu} \; T_{03} \tag{2}$$

<u>Stress parameter</u> for tapered blades (ref 5):

$$\Sigma \equiv \frac{\sigma_b}{\gamma \rho_b} = \frac{\omega^2 d^2}{2g} (L/d)$$

$$= \frac{\omega^2 A}{2\pi g} = 4(U^2/2g)(L/d) \qquad (3)$$

where σ_b is due to rotation only.

This stress parameter commonly runs, for stationary or heavy propulsion plants, at about:

$\Sigma_c = 7000$ ft.

$\Sigma_t = 9000$ ft.

while in portable and aircraft plants the figure

$\Sigma_t = 14,000$ ft.

is frequently reached.

An examination of the mechanical properties of presently available alloys reveals that at the moderate temperature level of compressors the operating stresses are limited to about 36,000 to 42,000 psi by yield strength and endurance limits. At the temperature level of turbines the operating stresses are limited by creep rate to a value roughly proportional to T_{03}^{-4}.

Approximation of the relation between static and stagnation densities:

$$\frac{wRT}{p} = \frac{w}{\rho} = \frac{w}{\rho_o} \cdot \frac{\rho_o}{\rho}$$

$$\frac{\rho_o}{\rho} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{1}{k-1}} \approx \left[1 + \frac{1}{2} M^2\right]$$

$$\frac{\rho_0}{\rho} = \left[1+\frac{k-1}{2}M^2\right]^{\frac{1}{k-1}} \cong \left[1+\frac{1}{2}M^2\right]$$

$$M^2 = \frac{C^2}{kgRT} \approx \frac{C_x^2}{kgRT_o}$$

By the continuity relation:

$$\frac{wRT}{P} = AC_x \approx \frac{wRT_o}{P_o}\left[1 + \frac{C_x^2/2g}{kRT_o}\right]$$

$$F_{m1} \equiv \frac{C_x^2/2g}{kRT_{o1}} \quad ; \quad F_{m3} \equiv \frac{C_x^2/2g}{kRT_{o3}} \qquad (4)$$

For the compressor discharge (or combustor inlet):

$$AC_{x_2} = \frac{wRT_{o2}}{P_{o2}}\left[1+F_{m_2}\right] = \frac{wRT_{o1}r_c^{\lambda}}{P_{o1}r_c}\left[1+r_c^{-\lambda} F_{m1}\right]$$

$$= \frac{wRT_{o1}}{P_{o1}r_c}\; r_c + F_{m1} \qquad (5)$$

A corresponding relation holds for the flow at turbine outlet, expressed in terms of T_{o3} .

Mean density of fluid in the compressor:

$$\frac{\rho_1}{\rho_m} = \frac{\rho_1}{1/2(\rho_1+\rho_2)} = \frac{2}{1+(\rho_2/\rho_1)} \approx \frac{2}{1+(\rho_{o2}/\rho_{o1})}$$

$$= \frac{2}{1+(P_{o2}/P_{o1})(T_{o1}/T_{o2})}$$

$$= \frac{2}{1+r_c^{1-\lambda}} \equiv F_c \qquad (6)$$

and correspondingly for the turbine:

$$\frac{\rho_4}{\rho_m} \approx \frac{2}{1+r_t^{\,1-\mu}} \equiv F_t \tag{6a}$$

These functions are obtainable from Fig. 2.

Rotational speed of a machine, optimized from the standpoint that the parts under radial stress are all at the allowable limit expressed by the stress parameter Σ, may be determined by combining (3) and (4) with the geometrical relations of the wheel to yield

$$\text{RPM} = \frac{60\,\omega}{2\pi} = 30\left[\frac{P(C_x/U)}{\pi w R T}\right]^{1/2}\left[\frac{2g^2\,\Sigma^2}{(L/d)}\right]$$

Since the longest blades and consequently the highest blade stresses are found at compressor inlet and turbine outlet, relating the general equation to these two stations gives, for the compressor:

$$\text{RPM}_c = 272\left[\frac{P_{o1}(C_x/U)_1}{wRT_{o1}(F_{m1}+1)}\right]^{1/2}\left[\frac{\Sigma_c^2}{(L/d)_1}\right]^{1/4} \tag{7}$$

and for the turbine:

$$\text{RPM}_t = 272\left[\frac{P_{o4}(C_x/U)_4}{wRT_{o4}(F_{m4}+r_t^{-\mu})}\right]^{1/2}\left[\frac{\Sigma_t^2}{(L/d)_4}\right]^{1/4} \tag{7a}$$

It will be noted, when considering possible plant cycles, that a wide change in pressure ratio causes at most a 10% change in optimum RPM directly, but rather affects it via the air rate or w/P^* ratio. Turbine inlet temperature affects permissible rotational speed slightly in a direct manner, but much more heavily via its bearing on the stress

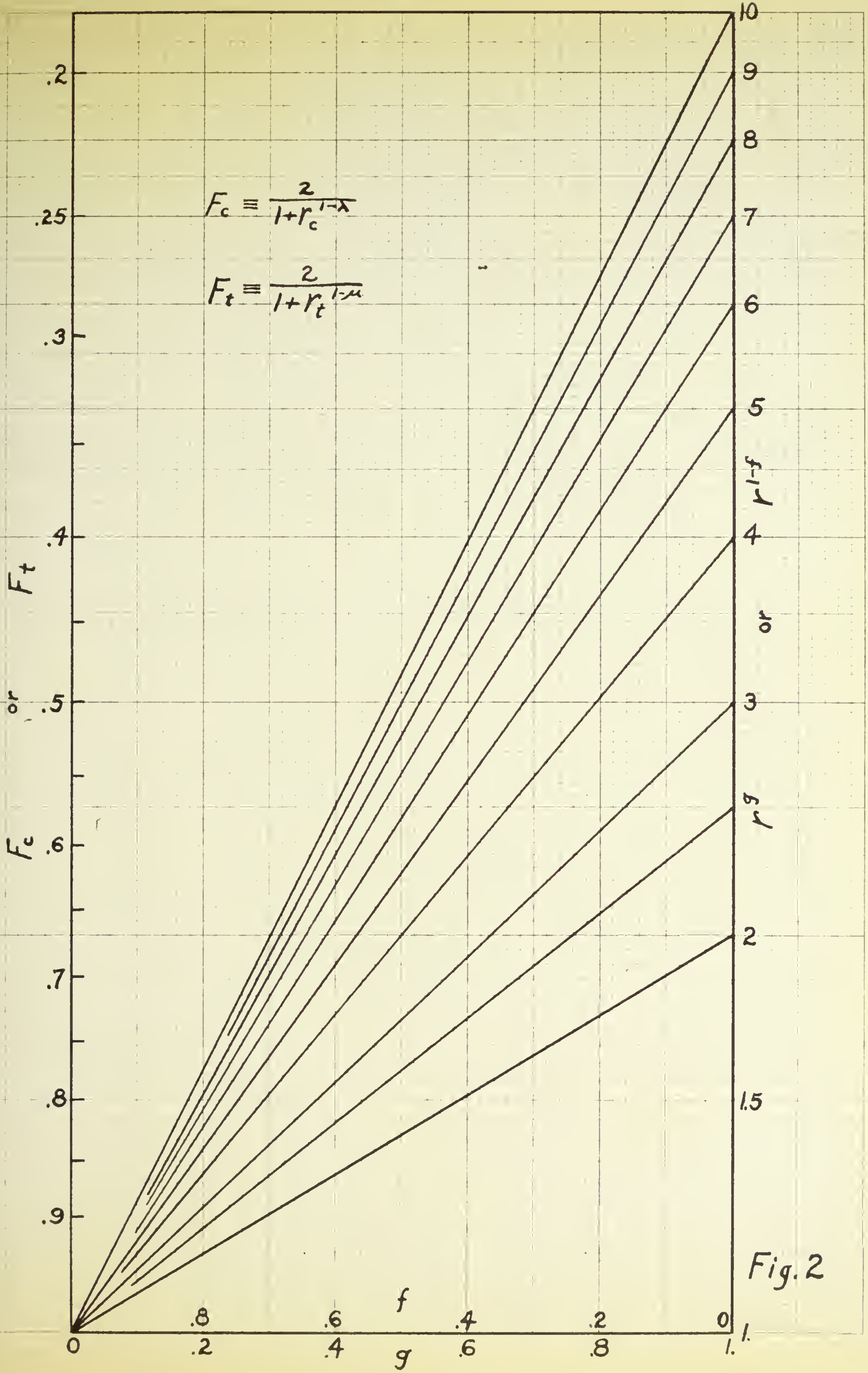

$F_c \equiv \frac{2}{1+r_c^{1-\lambda}}$
$F_t \equiv \frac{2}{1+r_t^{1-\mu}}$
F_c or F_t
r^g or r^{1-f}
f
g
.2
.25
.3
.4
.5
.6
.7
.8
.9
10
9
8
7
6
5
4
3
2
1.5
1.
0
.2
.4
.6
.8
1.

Fig. 2

parameter.

<u>Pitch diameter</u> of a rotor, optimized from the same standpoint as the RPM, is likewise determined to appear as:

$$d = \left[\frac{2wRT}{P(C_x/U)}\right]^{1/2} \left[2g\sum(L/d)\right]^{-1/4}$$

Relating this to compressor inlet and turbine outlet as before yields, for the compressor:

$$d_c = .282\left[\frac{wRT_{o1}}{P_{o1}(C_x/U)_1}\,(F_{m1}+1)\right]^{1/2} \left[\Sigma_c(L/d)_1\right]^{-1/4} \qquad (8)$$

$$D_c = \left[1+(L/d)_1\right] d_c \qquad (9)$$

and for the turbine:

$$d_t = .282\left[\frac{wRT_{o3}}{P_{o4}(C_x/U)_4}\,(F_{m3}+r_t^{-\mu})\right]^{1/2} \left[\Sigma_t(L/d)_4\right]^{-1/4} \qquad (8a)$$

$$D_t = \left[1+(L/d)_4\right] d_t \qquad (9a)$$

The same remarks regarding the effects of pressure ratio and turbine inlet temperature on the RPM apply generally to the diameter relations above, but of course in the reverse direction.

<u>Power requirement and output</u>, assuming a mean value of C_p and of k to be applicable over the temperature spans of each component, may be expressed, with (1) and (2), as:

$$P^* = w(h_1-h_2) \approx wC_p(T_{o1}-T_{o2})$$

which becomes, for the compressor:

$$P_c^* = wC_{pc}T_{o_1}(r_c^{\lambda} - 1) \qquad (10)$$

and for the turbine, with negligible leaving loss:

$$P_t^* = wC_{pt}T_{o_3}(1 - r_t^{-\mu}) \qquad (10a)$$

Allowing for leaving losses but assuming the leaving velocity nearly axial, or with negligible whirl, the last becomes:

$$P_t^* = w\left[C_{pt}T_{o_3}(1 - r_t^{-\mu}) - \frac{C_x^2}{2gJ}\right] \qquad (10b)$$

In a plant cycle the effect of pressure ratio on air rate, w/P*, is well known, and the curve of Fig. 4 is representative for a plant in which the output takes the form of shaft power. Since, knowing the cycle temperatures and the probable component efficiencies, the air rate is a known function of pressure ratio only, it will frequently be convenient to arrange an expression for, say, component weight in the form:

$$W^* = F(\text{cycle conditions}) \, . F(\text{pressure ratio})$$

Effect of Pressure Ratio on Component Size and Weight

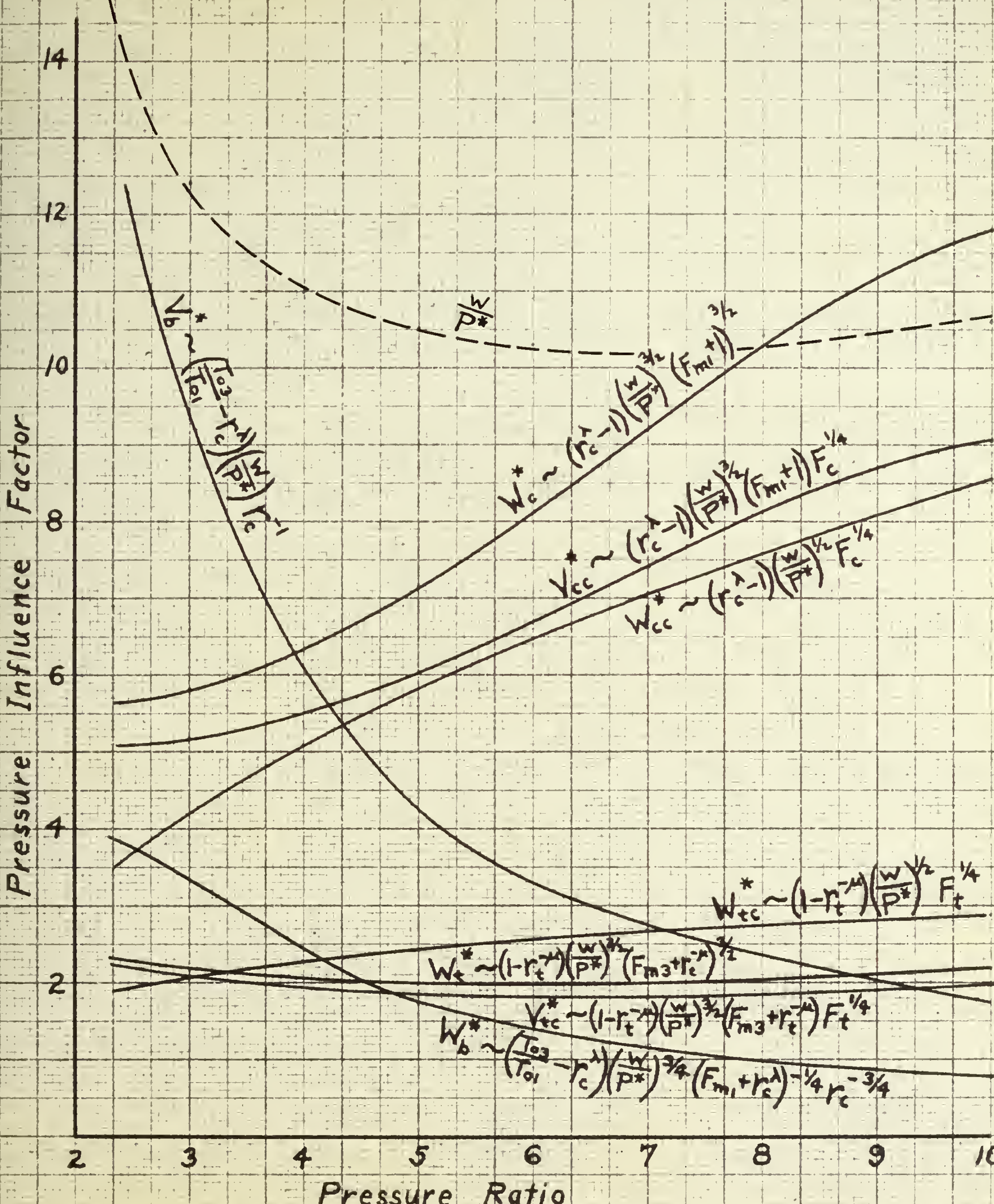


Fig. 4

III. MATCHING COMPONENTS

In matching components to form a plant cycle certain requirements are to be met, some of them necessary and obvious if the plant is to operate at all, others desirable and implicit if it is to give optimum satisfaction. For the simple cycle single-shaft jet propulsion plant (CBTJ), exemplified by aircraft installations for instance, the following hold:

$$RPM_c = RPM_t \qquad w_c = w_t$$

$$r_c P_{o_1} = r_t P_{o_4} \qquad R_c = R_t$$

$$C_{pc} < C_{pb} < C_{pt}$$

Taking the ratio of (8) to (8a) and using (3) to eliminate gives:

$$\frac{d_c}{d_t} = \left[\frac{r_c T_{o_1}(F_{m1}+1)(C_x/U)_4 (L/d)_4}{r_t T_{o_3}(F_{m3}+r_t^{-\mu})(C_x/U)_1 (L/d)_4}\right]^{1/3} \tag{11}$$

or:

$$\frac{d_c}{d_t} = \left[\frac{r_c T_{o_1}(F_{m1}+1)(C_x/U)_4\ \sigma_{bt}}{r_t\ T_{o_3}(F_{m3}+r_t^{-\mu})(C_x/U)_1\ \sigma_{bc}}\right] \tag{11a}$$

the latter holding only if $\mathcal{T}$ is the same for both machines, and the blade material is such that ρ_b is the same for both. Since turbine and compressor power are substantially equal at all times (10) and (10a) combine to give:

$$C_{pc} T_{o_1}(r_c^{\lambda}-1) = C_{pt} T_{o_3}(1-r_t^{-\mu})$$

The useful power output of such a plant takes the form of thrust, dependent on jet velocity.

$$\frac{C_j^2}{2gJ} = C_{pb}(T_{o4} - T_j) = C_{pb}T_{o4}\left[1 - \frac{T_j}{T_{o4}}\right]$$

$$C_j^2 = 2gJC_{pb}T_{o3}\left[r_t^{-\mu} - r_c^{-\nu}\, r_t^{\nu-\mu}\right]$$

or in terms of the compressor pressure ratio only:

$$C_j^2 = 2gJC_{pb}T_{o3}\left[1 - \frac{C_{pc}T_{o1}}{C_{pt}T_{o3}}\,(r_c^{\lambda}-1)\right]\left[1 - \left(1 - \frac{C_{pc}T_{o1}}{C_{pt}T_{o3}}(r_c^{\lambda}-1)\right)^{-1/\mu} r_c^{-\nu}\right] \tag{12}$$

In the special case of static thrust:

$$\frac{F_s}{W} = \frac{C_j}{g}$$

For the simple cycle stationary or heavy propulsion plant (CBTG), exemplified by the railway locomotive unit, the following hold:

$G \cdot RPM_c = RPM_t$ where G is a gear ratio

$W_c = W_t$ $\qquad$ $R_c = R_t$

$P_{o1} = P_{o4}$ $\qquad$ $r_c = r_t$

$C_{pc} < C_{pb} < C_{pt}$

Substituting (7) and (7a) into the first of these relations, and making use of the others where appropriate:

$$G^2 = \frac{T_{o1}(F_{m1}+1)(C_x/U)_4}{T_{o3}(F_{m3}+r^{-\mu})(C_x/U)_1}\left[\frac{(L/d)_1\,\Sigma_t^2}{(L/d)_4\,\Sigma_c^3}\right]^{1/2} \tag{14}$$

or, using (3) to eliminate Σ leaves:

$$G = \frac{T_{o2}(F_{m2}+r^{-\mu})(C_x/U)_1(L/d)_1 d_c^2}{T_{o1}(F_{m1}+1)(C_x/U)_2(L/d)_2 d_t^2} \tag{15}$$

The net power output of this type of plant is:

$$P^*_n = P^*_t - P^*_c$$

By (10) and (10a) then:

$$\frac{P^*}{w} = JC_{pc}T_{o1}\left[\frac{C_{pt}}{C_{pc}}\,\frac{T_{o2}}{T_{o1}}\,(1-r^{-\mu})-(r^{\lambda}-1)\right] \tag{16}$$

And the cycle thermal efficiency becomes:

$$\eta = \frac{C_{pt}(T_{o2}/T_{o1})(1-r^{-\mu})-C_{pc}(r^{\lambda}-1)}{C_{pb}\left[(T_{o2}/T_{o1})-r^{\lambda}\right]} \tag{17}$$

IV. DETERMINATION OF THE STAGE

Thus far, no consideration has been given to aerodynamic relations or the factors affecting efficiency within a stage, nor with stage dimensions. There are to be developed an expression for the axial width of the blade row, the blade spacing, and the number of stages required. The length and volume of machine rotors is readily obtained then from these expressions and those previously set forth.

Zweifel (ref. 1) has developed an aerodynamic load coefficient

$$\Psi_t = 2\sin^2\beta_2(\cot\beta_2 - \cot\beta_1)\frac{1}{\sigma}$$

based upon the attainable pressure distribution around an airfoil, which coefficient he shows to have a value very near eight-tenths for minimum pressure loss and minimum drag/lift ratio in a cascade. This significantly corresponds to the preferred design deflection angle of eight-tenths that for which stall occurs, presented by Howell (ref. 2). The work done in a stage may be written as:

$$\frac{W}{U^2/2g} = 2(C_x/U)(\cot\beta_2 - \cot\beta_1)$$

Combining the above two expressions, the optimum solidity of a blade row may be stated as:

$$\sigma = 1.25\,\frac{\sin^2\beta_2}{(C_x/U)}\;\frac{W}{U^2/2g} \qquad (18)$$

On the other hand, Schnittger (ref. 3) has indicated that for optimum stage efficiency the camber of a blade is related

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Zweifel (ref. 1) has developed an aerodynamic load coefficient:

$$\psi_t = 2\sin^2\beta_2(\cot\beta_2-\cot\beta_1)\frac{t}{b}$$

based upon the attainable pressure distribution around an airfoil, which coefficient he shows to have a value very near eight-tenths for minimum pressure loss and minimum drag/lift ratio in a cascade. This significantly corresponds to the preferred design deflection angle of eight-tenths that for which stall occurs, presented by Howell (ref. 2). The work done in a stage may be written as:

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On the other hand, Schlichting (ref. 3) has indicated that for optimum stage efficiency the camber of a blade is related

to the gas leaving angle by:

$$.5 < \theta/\beta_2 < .55$$

and further that the fluid deflection angle in decelerating cascades is related to this leaving angle by:

$$\varepsilon = .307\beta_2(c/s)^{1/2} \approx .307\beta_2\phi^{1/2}$$

Hence, by combination it follows that:

$$\beta_1 = \beta_2\,(1-.307\phi^{1/2})$$

and by substitution in Zweifel's relation above, the optimum solidity becomes:

$$\phi = 2.5\sin^2\beta_2\left[\cot\beta_2-\cot(1-.307\phi^{1/2})\beta_2\right] \qquad (19)$$

which is readily solved by trial, on the first attempt estimating $\phi = 1$ on the right.

Figures 5 and 6 illustrate the nomenclature used above. The latter is a reproduction of the curves presented by Zweifel on which has been drawn the line of maximum turbine stage efficiency given by Hawthorne (ref. 4). The difference in notation between these figures and that used herein should be noted.

Gas bending stress in a blade may be found from the fundamental formula of mechanics:

$$\sigma_g = \frac{my}{I} = \frac{m}{I/y}$$

wherein the moment is:

$$m = 1/2\;LF = 1/2\;d(L/d)\;\frac{wW_2}{ZU}$$

$$= \frac{w}{Z}\,(L/d)\,\frac{W}{U^2/2g}\,(U^2/2g)$$

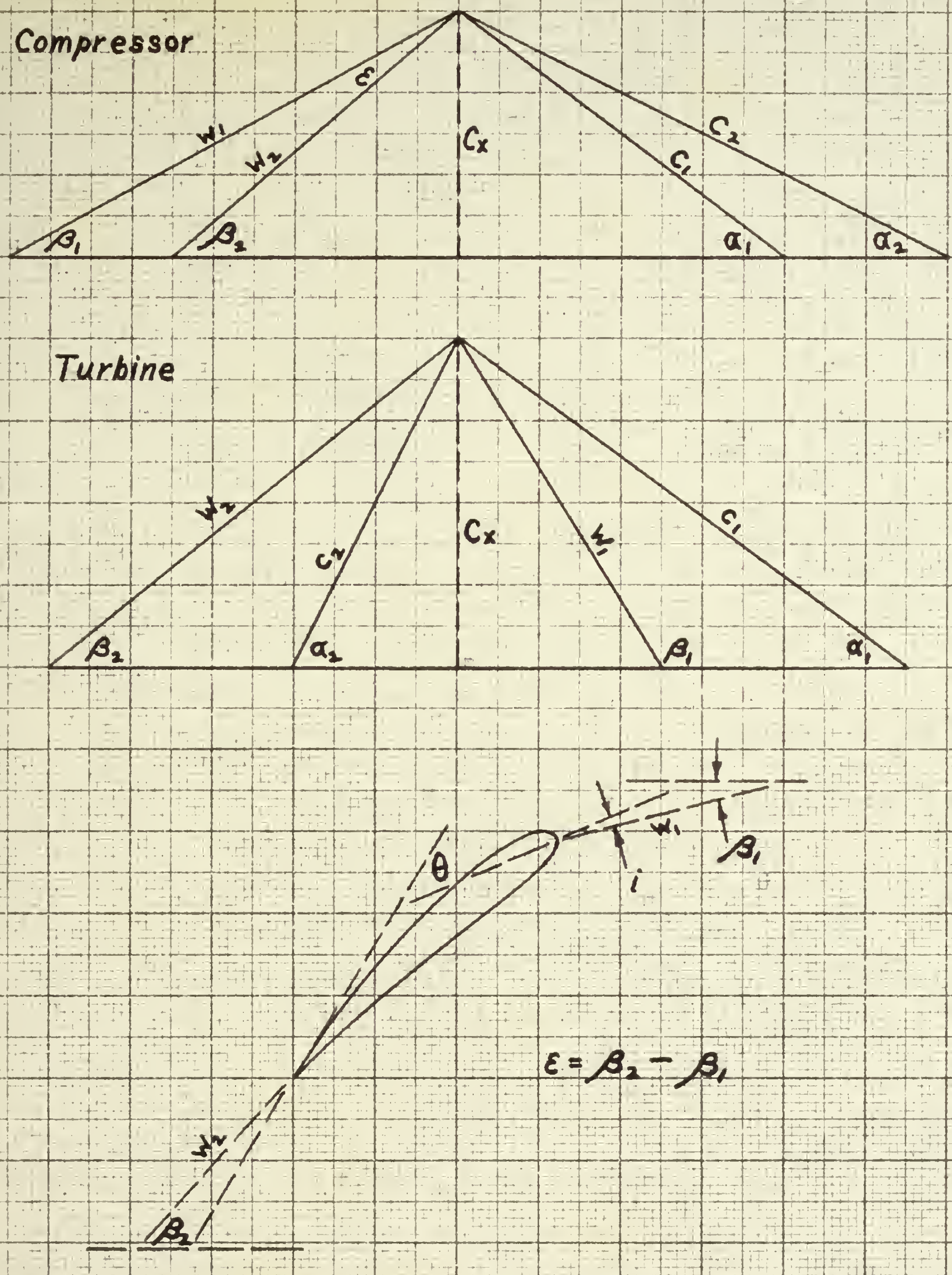

Compressor
ε
W_1
W_2
C_x
C_2
C_1
β_1
β_2
α_1
α_2
Turbine
W_2
C_2
C_x
W_1
C_1
β_2
α_2
β_1
α_1
θ
W_1
β_1
i
$\varepsilon = \beta_2 - \beta_1$
W_2
β_2

Fig. 5

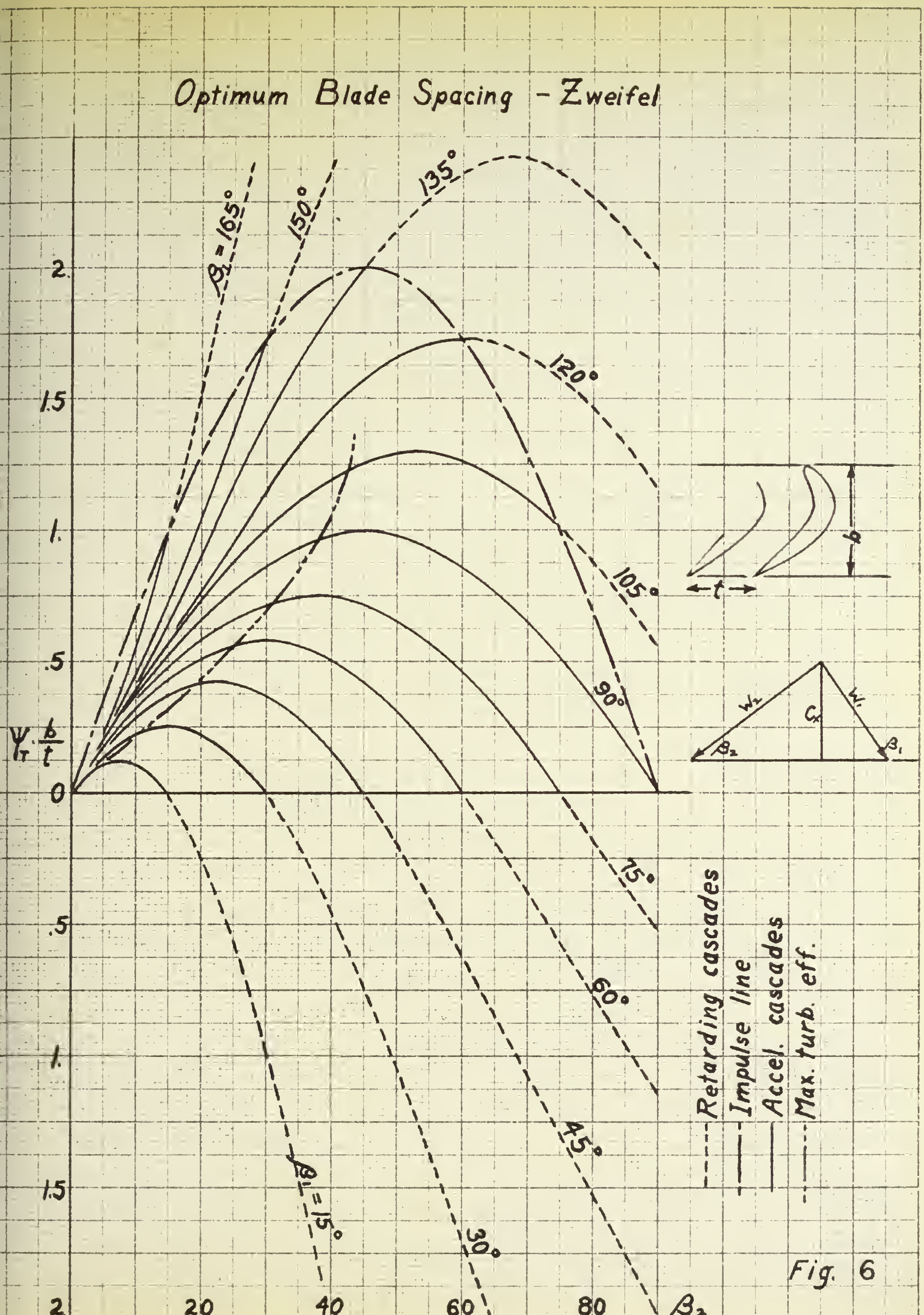

Optimum Blade Spacing - Zweifel
$\beta_2 = 165°$
150°
135°
120°
105°
90°
75°
60°
45°
30°
$\beta_1 = 15°$
$\Psi_T \cdot \frac{b}{t}$
2.
1.5
1.
.5
0
.5
1.
1.5
2
20
40
60
80
β_2
b
t
W_2
C_x
W_1
β_2
β_1
Retarding cascades
Impulse line
Accel. cascades
Max. turb. eff.

Fig. 6

But from the geometry, and (18) above:

$$Z = \frac{\pi \not{\delta}\, \delta}{(L/d)} = \frac{1.25\, \pi\, \delta \sin^2 \beta_2}{(L/d)(C_x/U)} \quad \frac{W}{U^2/2g}$$

Hence, by substitution:

$$m = \frac{w(L/d)^2 (C_x/U)(U^2/2g)}{1.25\pi\, \omega\, \delta \sin^2 \beta_2}$$

In order to evaluate the blade section modulus, I/y, various airfoil section plans were measured to determine the effect of camber and thickness ratio in the moment of inertia and extreme fiber distance. A standard thickness distribution was used, with camber varied 25° to 109°, thickness ratio varied 7% to 10%, based on a parabolic camber line. The extreme of the shapes so measured are shown in Fig. 3. From this there was deduced (θ in radians here):

$$\frac{I}{y} = \frac{b^3}{1000}\left[.35\theta^{2.4} - 85t^2\right] \qquad (20)$$

Substituting this in the above:

$$\sigma_g = \frac{2280\ w\ (L/d)^2 (C_x/U)(U^2/2g)}{\pi\, \omega\, \delta\, b^2 \sin^2 \beta_2\, F_b} \qquad (21)$$

wherein

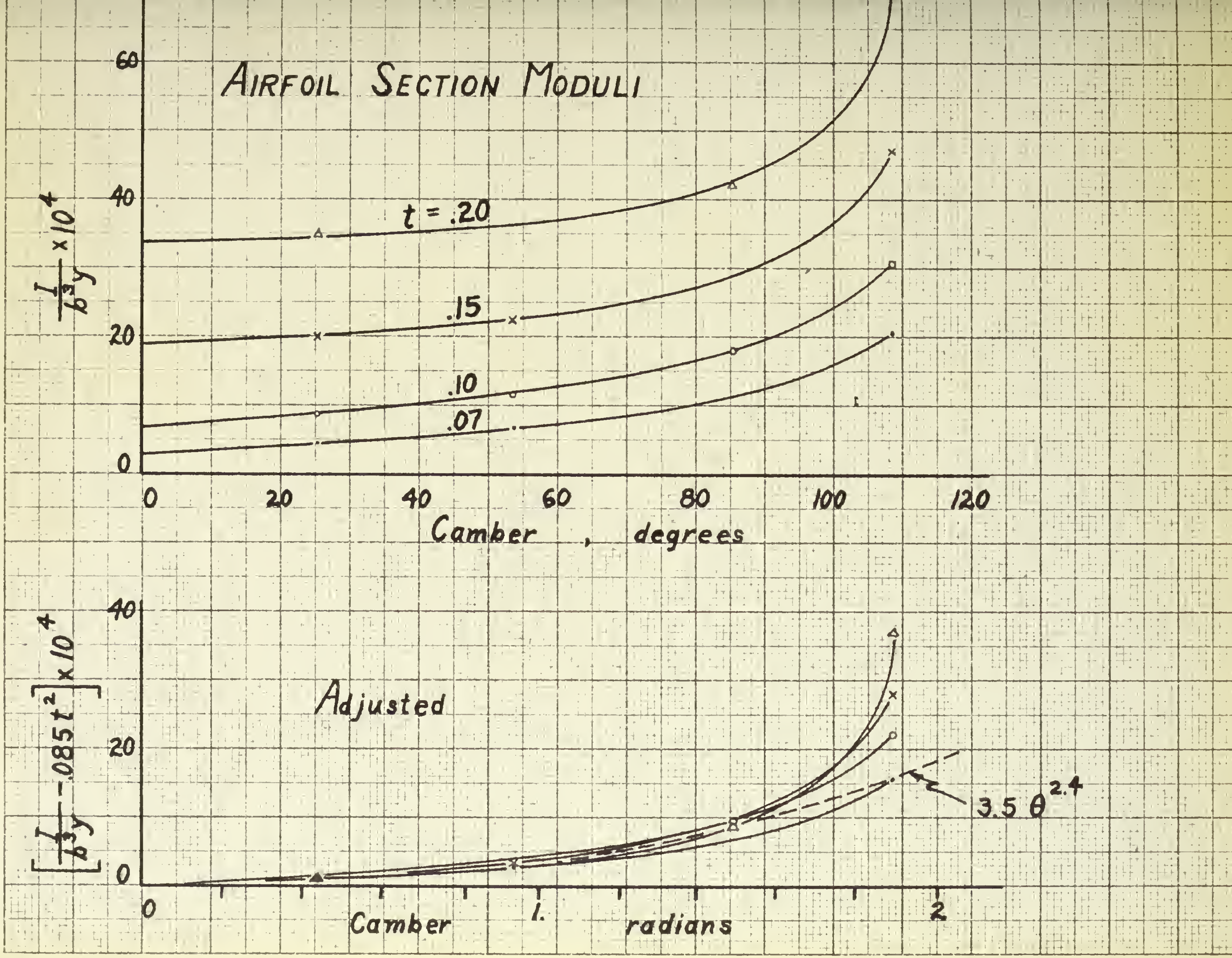
$$F_b \equiv \theta^{2.4} - 242.5t^2$$

Replacing the aspect ratio by its definition:

$$\delta \equiv (L/b) = (L/d)d/b$$

and using (3) to eliminate ω, d and $(U^2/2g)$:

$$b^3 = \frac{2280}{4\pi\sqrt{2g}} \quad \frac{w(L/d)^{1/2} (C_x/U)^{1/2}}{\sigma_g F_b \sin^2 \beta_2}$$


AIRFOIL SECTION MODULI
$\frac{I}{b^3 y} \times 10^4$
60
40
20
0
$t = .20$
.15
.10
.07
0
20
40
60
80
100
120
Camber , degrees
40
20
0
$\left[\frac{I}{b^3 y} - .085 t^2\right] \times 10^4$
Adjusted
$3.5\,\theta^{2.4}$
0
1.
2
Camber radians
Fig. 7

K & E CO., N. Y.

wherefrom:

$$b = \frac{4.75}{\sin \beta_2} \left[\frac{w(C_x/U)}{\sigma_g F_b} \right]^{1/2} \left[(L/d)\Sigma \right]^{1/4}$$

Since in obtaining the overall rotor length the mean stage width is desired, a mean value of (L/d) must be obtained for use in the foregoing, and (L/d) in turn is proportional to local density if C_x is to be maintained constant. Hence, by (6), for the compressor:

$$(L/d)_{mc} = (L/d)_1(\rho_1/\rho_m) = (L/d)_1 F_t \tag{22}$$

and by (6a), for the turbine:

$$(L/d)_{mt} = (L/d)_4 F_t \tag{22a}$$

These then lead to the compressor row width:

$$b_{mc} = \frac{4.75}{\sin\beta_2} \left[\frac{w(C_x/U)_1}{\sigma_g F_{bc}} \right]^{1/2} \left[(L/d)_1 F_c \Sigma_c \right]^{1/4} \tag{23}$$

and the turbine's:

$$b_{mt} = \frac{4.75}{\sin\beta_2} \left[\frac{w(C_x/U)}{\sigma_g F_{bt}} \right]^{1/2} \left[(L/d)_4 F_t \Sigma_t \right]^{1/4} \tag{23a}$$

The number of stages required depends jointly on the work to be done in a machine and the amount of work efficiently attainable per stage, or:

$$N = \frac{\Delta h}{W_s \eta_{sc}}$$

This becomes, for the compressor:

$$N_c = \frac{J\, C_{pc}\, T_{o1}(r_c^{\lambda}-1)}{\frac{W}{U^2/2g}\ \frac{U^2}{2g}\ \eta_{sc}}$$

and by using (3) to eliminate circumferential speed:

$$N_c = \frac{3112 C_{pc} T_{o1}(r_c^{\lambda}-1)(L/d)_1}{\Sigma_c\ \eta_{sc}\ \frac{W}{U^2/2g}} \tag{24}$$

Correspondingly, the number of stages for the turbine is found to be:

$$N_t = \frac{3112 C_{pt} T_{o3}(1-r_t^{-\mu})\ \eta_{st}(L/d)_4}{\Sigma_t\ \frac{W}{U^2/2g}} \tag{24a}$$

General experience has shown that the blade aspect ratio, $\delta \equiv L/b$, is in practice limited at the lower extreme by tip losses and wall friction, and at the upper extreme by stress considerations and mounting secondary flow losses. In view of these limits, the aspect ratio may be taken to vary from one to five, with the optimum at possibly two or three for "good practice".

By inserting (8) and (23) in the definition, aspect ratio may be related to other quantities, however, and the result will serve, in combination with the rule-of-thumb above, to limit the range of variation of other parameters. Thus, for the compressor:

$$\delta_{mc} = .0594\ \frac{\sin \beta_2}{(C_x/U)_1 F_c^{1/4}} \left[\frac{\sigma_g R T_{o1}(F_{m1}+1) F_{bc}(L/d)_1}{P_{o1}\ \Sigma_c}\right]^{-1/2} \tag{25}$$

$$N_c = \frac{J\, C_{pa}\, T_{01}\, (r_c^{\lambda}-1)}{\frac{\psi}{U^2/2g}\,\frac{U^2}{2g}\,\eta_{sc}}$$

and by using (1) to eliminate circumferential speed:

$$N_c = \frac{5112\, C_{pc}\, T_{01}\, (r_c^{\lambda}-1)(L/S)_1}{\Sigma_c\, \eta_{sc}\, \frac{\psi}{U^2/2g}} \qquad (34)$$

Correspondingly, the number of stages for the turbine is found to be:

$$N_t = \frac{5112\, C_{pt}\, T_{05}\, (1-r_t^{-\lambda'})\, \eta_{st}\, (L/S)_t}{\Sigma_t\, \frac{\psi}{U^2/2g}} \qquad (35)$$

General experience has shown that the blade aspect ratio, $\delta = L/b$, is in practice limited at the lower extreme by tip losses and wall friction, and at the upper extreme by stress considerations and secondary flow losses. In view of these limits, the aspect ratio may be taken to vary from one to five, with the optimum at possibly two or three for 'good practice'.

By inserting (3) and (25) in the definition, aspect ratio may be related to other quantities, however, and the result will serve, in combination with the rule-of-thumb above, to limit the range of variation of other parameters. Thus, for the compressor:

$$\delta_{sc} = .0984\, \frac{(C_x/U)_1\, r_c^{1/2}}{\sin\beta_3} \left[\frac{U_R\, \eta_{sc}\, (r_{sc}+1)\, T_{0x}\, (L/S)_1}{\Sigma_c \qquad r_{01}}\right]^{1/2} \qquad (36)$$

and for the turbine:

$$\delta_{mt} = .0594 \frac{\sin \beta_3}{(C_x/U)_4 F_t^{1/4}} \left[\frac{\sigma_g R T_{03} (F_{m3} + r_t^{-1/4}) F_{bt} (L/d)_4}{P_{04} \; \Sigma_t} \right]^{1/2} \qquad (25a)$$

V. ROTOR SIZE AND WEIGHT

Rotor size is controlled by the number of stages, the axial length of each, and the tip diameter of the blading. For the length:

$$L^* = 2Y \cdot b_m \cdot N$$

wherein the clearance ratio between adjacent rows is indicated by Y. From (23) and (24), and by properly arranging terms, it may be shown that for the compressor:

$$L^*_c = 29,600 \; \frac{C_{pc} T_{o1} (C_x/U)_1^{\frac{1}{2}}}{\frac{W}{U^2/2g} \; \eta_{sc}} \left[\frac{Y (L/d)_1}{\sin\beta_2 \, \sigma_g^{\frac{1}{2}} \, \Sigma_c^{\frac{3}{4}} \, F_{bc}^{\frac{1}{2}}} \right] \left[\left(\frac{W}{P^*}\right)^{\frac{1}{2}} F_c^{\frac{1}{4}} (r_c^{\lambda} - 1) \right] P^{*\frac{1}{2}} \tag{26}$$

and for the turbine:

$$L^*_t = 29,600 \; \frac{C_{pt} T_{o2} (C_x/U)_4 \, \eta_{st}}{\frac{W}{U^2/2g}} \left[\frac{Y (L/d)_4}{\sigma_g^{\frac{1}{2}} \, \Sigma_t^{\frac{3}{4}} F_{bt}^{\frac{1}{2}} \, \sin\beta_2} \right] \left[\left(\frac{W}{P^*}\right)^{\frac{1}{2}} F_t^{\frac{1}{4}} (1 - r_t^{-\mu}) \right] P^{*\frac{1}{2}} \tag{26a}$$

In each of the above, the final bracket represents the explicit influence of pressure ratio while the first incorporates the influence of cascade geometry, broadly speaking. The trend of this pressure ratio influence is shown in Fig. 4 for compressor and turbine separately.

Rotor volume may be stated simply as:

$$V = \frac{1}{4} \pi D^2 L^*$$

Substituting from (9) and (26) and rearranging as for the length yields, for the compressor:

V. ROTOR SIZE AND WEIGHT

Rotor size is controlled by the number of stages, the axial length of each, and the tip diameter of the blading. For the length:

$$L^* = Cl \cdot b_x \cdot S$$

whereas the clearance ratio between adjacent rows is indicated by S. From (23) and (24), and by properly arranging terms, it may be shown that for the compressor:

$$L^*_c = 20,000 \frac{C_{ax} Cos(C_x/U)_1^{\frac{1}{2}}}{\frac{1}{N\sqrt{\theta}} \sqrt{\theta e}} \left[\frac{\Sigma(b/s)_1 -}{\sin\beta_m G_x^{\frac{1}{2}} \Sigma_c^{3/4} \eta_{sc}^{1/2}}\right] \left[\left(\frac{W}{p\sigma}\right)^{\frac{1}{2}} T_0^{\frac{1}{4}}(\pi_c^{k}-1)\right] \eta^{\frac{1}{2}}$$

(28)

and for the turbine:

$$L^*_t = 25,000 \frac{C_{xt} Cos(C_x/U)_1 \Sigma_{xt}(b/s)_1}{\frac{1}{N\sqrt{\theta}}} \left[\frac{}{G_x^{\frac{1}{2}} \Sigma_t^{3/4} \eta_{st}^{1/2} \sin\beta_m}\right] \left[\left(\frac{W}{p\sigma}\right)^{\frac{1}{2}} T_3^{\frac{1}{4}}(1-\pi_t^{-k})\right] \eta^{\frac{1}{2}}$$

(28a)

In each of the above, the final bracket represents the explicit influence of pressure ratio while the first incorporates the influence of cascade geometry, broadly speaking. The trend of this pressure ratio influence is shown in Fig. 4 for compressor and turbine separately.

Rotor volume may be stated simply as:

$$V = \frac{\pi}{4} D^2 L^*$$

Substituting from (4) and (24) and rearranging as for the length yields, for the compressor:

$$V_c = \frac{1850\, R C_{pc} T_{o1}^{2}\, P^{*\,3/2}}{\eta_{sc} \dfrac{W}{U^2/2g} P_{o1} (C_x/U)_1^{1/2}} \left[\frac{Y\left(1-(L/d)_1\right)^2 (L/d)_1^{3/4}}{\sin\beta_2\, \sigma_g^{\frac{1}{2}}\ F_{bc}^{1/2}\ \Sigma_c^{5/4}}\right]$$

$$\cdot \left[(F_{m1}+1)(r_c^{\lambda}-1)F_c^{1/4}\ \left(\frac{W}{P^*}\right)^{3/2}\right] \qquad (27)$$

and likewise for the turbine:

$$V_t = \frac{1850\, R C_{pt} T_{o2}^{2}\, P^{*\,3/2}\ \eta_{st}}{\dfrac{W}{U^2/2g} P_{o4} (C_x/U)_4^{1/2}} \left[\frac{Y\left(1+(L/d)_1\right)^2 (L/d)_4^{3/4}}{\sigma_g^{\frac{1}{2}}\ F_{bt}^{\frac{1}{2}}\ \sin\beta_2\ \Sigma_t^{5/4}}\right]$$

$$\cdot \left[(F_{m2}+r_t^{-\mu})(1-r_t^{-\mu})F_t\ \left(\frac{W}{P^*}\right)^{\frac{3}{2}}\right] \qquad (27a)$$

The two brackets here, as in the length relations, are intended to indicate the influence of stage geometry and pressure ratio respectively. The latter are shown in Fig. 4.

<u>Rotor weight</u> has been investigated for wheels of the DeLaval type by LaValle and Huppert (ref. 5). Their results, with some simplifying assumptions to fix a few minor influence variables which have small ranges, are here applied to both turbine and compressor, since the latter is taken to be of disk type construction and therefore comparable in each stage to a DeLaval wheel. All this is predicated on having a maximum allowable blade stress, as expressed by (3).

For one wheel or stage the weight is then:

$$W^*_s = (\text{disk} + \text{rim} + \text{blades})$$

$$= \frac{\pi (L/d)^3 d^3 \rho_b}{\delta} \left[\left(\frac{.55 t \not{z}}{(L/d)^3 \Delta_2} + \frac{\rho_d \Delta_1^2}{\rho_b \Delta_2} \right) \Delta_3 + \frac{\rho_d \Delta_1}{\rho_b \delta} + \frac{.55 t \not{z}}{(L/d)} \right] \tag{28}$$

wherein:

$$\Delta_1 \equiv \left[\frac{1}{(L/d)} - 1 - \frac{1}{\delta} \right]$$

$$\Delta_2 \equiv \left[\frac{1}{(L/d)} - 1 - \frac{2}{\delta} \right]$$

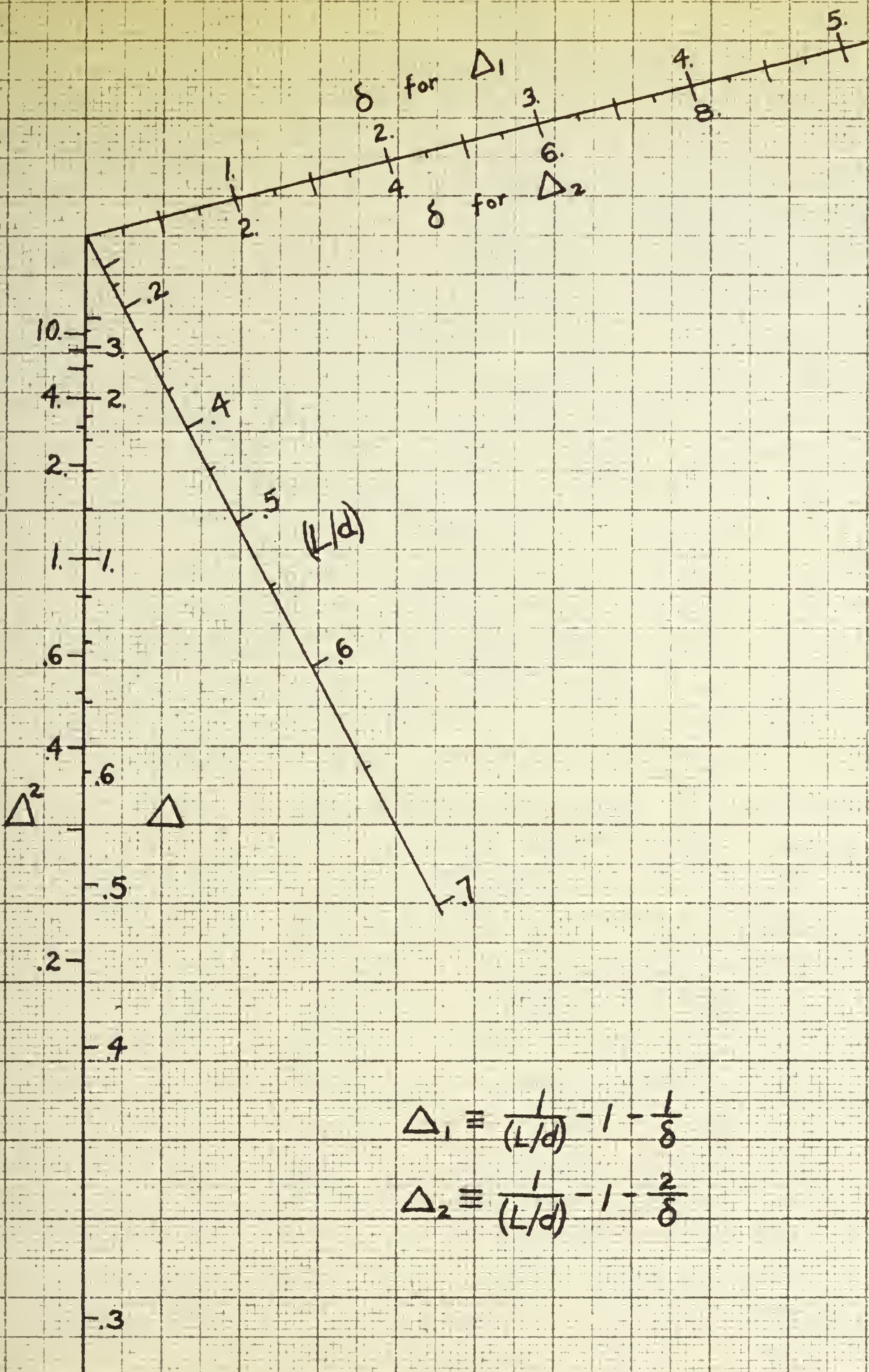
$$\Delta_3 \equiv \exp \left[.3 \frac{\rho_b}{\rho_d} \Delta_2^2 (L/d) \right] - 1$$

From Fig. 8 may be obtained Δ_1 and Δ_2.

The material density ratio, (ρ_b/ρ_d), will approach unity in the case of a turbine or in the case of a compressor not having light metal blades. With this additional simplification (28) is combined with (8) and (24) to yield:

$$W^*_c = \frac{70\, C_{pc} T_{o1}}{\eta_{sc} \frac{W}{U^2/2g} \sum_c^{7/4}} \left[\frac{P^* R T_{o1}}{P_{o1} (C_x/U)_1} \right]^{3/2} \left[(L/d)_1^{1/4} \frac{W^*_s}{d^3} \right]$$

$$\cdot \left[(r_c^{\lambda} - 1) \left(\frac{W}{P^*} \right)^{3/2} (F_{m1} + 1)^{3/2} \right] \tag{28a}$$



$$\Delta_1 \equiv \frac{1}{(L/d)} - 1 - \frac{1}{\delta}$$

$$\Delta_2 \equiv \frac{1}{(L/d)} - 1 - \frac{2}{\delta}$$

Fig. 8

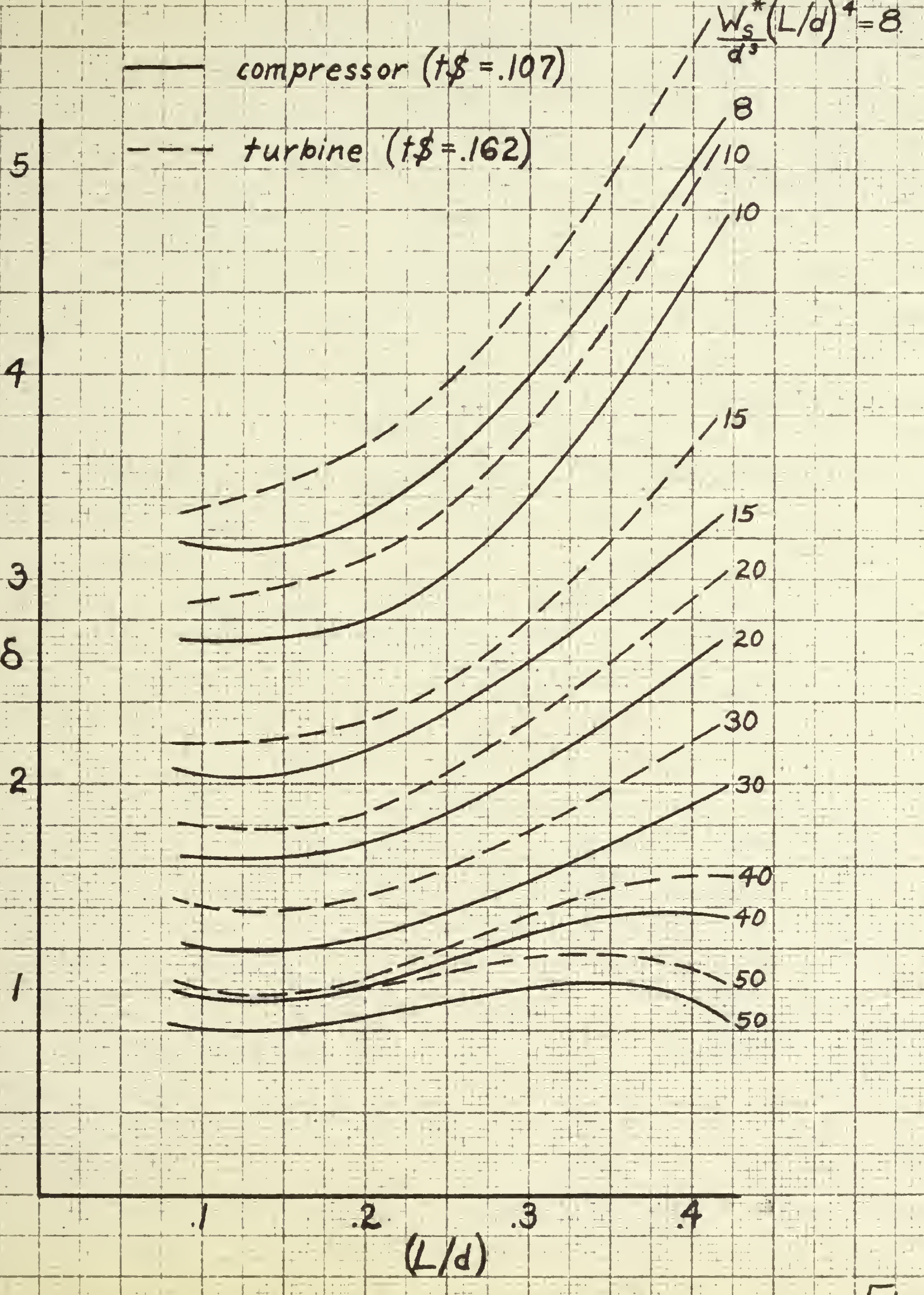
and:

$$W^*_t = \frac{70\ C_{pt} T_{c3}\ \eta_{st}}{\frac{W}{U^2/2g} \Sigma_t^{7/4}} \left[\frac{P^* R T_{03}}{P_{04} (C_x/U)_4}\right]^{3/2} \left[(L/d)_4^{1/4}\ \frac{W^*_s}{d^3}\right]$$

$$\cdot \left[(1-r_t^{-\mu})\ \left(\frac{W}{p^*}\right)^{3/2}\ (F_{m3}+r_t^{-\mu})^{3/2}\right] \qquad (28b)$$

Again the parts have been arranged so that the final bracket in each equation represents the explicit pressure ratio influence and the bracket immediately preceding represents the stage geometry. The pressure ratio factor is shown in Fig. 4 and the geometry factor, calculated from (28) above, appears in Fig. 9.

Due to its effect on rotor thickness, increasing aspect ratio causes a decrease in rotor weight, but a simultaneous rise in rotor volume. This last effect will appear later (see p. 43, eq. (33) et seq.). Hence the mean density of the rotor bulk drops doubly fast with rising aspect ratio. Since a high machine density tends toward compactness of plant for a fixed power output, this gives the first indication that low blade aspect ratios are to be preferred.



compressor (t$ = .107)
turbine (t$ = .162)
$\frac{W_s^*}{d^3}(L/d)^{\frac{1}{4}} = 8$
8
10
10
15
15
20
20
30
30
40
40
50
50
5
4
3
δ
2
1
.1
.2
.3
.4
(L/d)

Fig. 9

VI. COMBUSTION COMPONENTS AND CASINGS

Over the variety of combustion turbine plant types, from heavy stationary generating equipment to light weight air-craft propulsion sets, combustion equipment varies in weight and size fully as much as does any other component, and with considerably greater empiricism. Watson and Clarke (ref. 6) have summarized current practice. The first parameter to be considered is heat release rate per unit volume:

$$\frac{h_f w_f}{V_b P_{o2}} = \frac{w C_{pb}(T_{o3}-T_{o2})}{V_b r_c P_{o1}} = .65 \frac{BTU}{\text{sec. lb. ft.}}$$

The constant employed is based on the fact that good combustor performance regularly can be attained for heat release rates up to but not much exceeding 5.7×10^6 BTU/hr. ft.^{3} atm. The actual limit depends on a balance between heat conduction, diffusion, wall cooling and metallurgical properties. A slightly more conservative figure, 4.95, is used above.

Another limitation is flame stability in the moving gas flow. Based only on experience and good practice again, the permissible maximum bulk velocity entering the flame zone is found to be about 500 ft./sec. Hence, leaving again a margin of safety by using 400, there is:

$$\frac{w}{\rho_2 A_b} = 400 = \frac{wRT_{o1}\,(F_{m1}+r_c{}^{\lambda})}{A_b r_c P_{o1}} \quad \frac{\text{ft.}}{\text{sec.}}$$

Combustor length is then, assuming a shape approximately prismatic:

VI. COMBUSTION COMPONENTS AND CASINGS

Over the variety of combustion turbine plant types, from heavy stationary generating equipment to light weight air-craft propulsion sets, combustion equipment varies in weight and size fully as much as does any other component, and with considerably greater restriction. Watson and Clarke (ref. 5) have summarized current practice. The first parameter to be considered is heat release rate per unit volume:

$$\frac{Q_f F}{V_c p_{03}} = \frac{w_{gb}(T_{04}-T_{03})}{V_c p_{03}} = .85 \; \frac{BTU}{sec.\ lb.\ ft.}$$

The constant employed is based on the fact that good combustor performance regularly can be attained for heat release rates up to but not much exceeding 2.7×10^6 BTU/hr. ft.^{3} atm. The actual limit depends on a balance between heat convection, diffusion, wall cooling and metallurgical properties. A slightly more conservative figure, 4.92, is used above.

Another limitation is flame stability in the moving gas flow. Based only on experience and good practice again, the permissible maximum bulk velocity entering the flame zone is found to be about 500 ft./sec. Hence, leaving again a margin of safety by using 400, there is:

$$\frac{w}{\rho_3 A_b} = 400 = \frac{w_{gb} (T_{03}+T_{04})}{A_b p_{03}} \; \frac{ft.}{sec.}$$

Combustor length is then, assuming a shape approximately prismat

$$L_b^* = \frac{V_b}{A_b} = 616 \quad \frac{C_{pb}\left[(T_{o3}/T_{o1})-r_c^{\lambda}\right]}{R(F_{m1}+r_c^{\lambda})} \quad \text{ft.} \qquad (29)$$

In case, as is true for aircraft and certain other plants, combustion is to be equally shared among several chambers, a third limiting factor to be considered is minimum flame tube diameter. This may be met by: (a) limiting the number of separate chambers; or (b) increasing the total cross-sectional area beyond that required for permissible maximum gas velocity.

The weight of a combustor is a function of wall construction and surface area. Assuming that if a number of chambers are operated in parallel they are all exactly alike, then the total surface area is:

$$A_s = \pi d_b L^* N = 2\, V_b\, (\pi N/A_b)^{1/2}$$

$$= 109\, C_{pb}\left[\frac{T_{o3}}{T_{o1}} - r_s^{\lambda}\right]\left[\frac{NwT_{o1}}{Rr_c P_{o1}(F_{m1}+r_c^{\lambda}\;)}\right]^{1/2} \quad \text{ft.}^2$$

Wall construction is considered to be based on resistance to sagging and buckling rather than to rupture in tension. Thus for a cylinder in transverse loading, as a first approximation,

$$\sigma = \frac{my}{I} \sim \frac{(t.d)d}{(t.d)^3} \sim \frac{d}{(t.d)^2}$$

$$W_b^* \sim A_s\,(t.d) \sim A_s (d/\sigma\,)^{1/2} = A_s d/(\sigma\, d)^{1/2}$$

But since

$A_b \sim d^2 N$ and $V_b \sim d^2 NL^*$ there follows:

$$W_b^* \sim \frac{V_b}{(\sigma d)^{1/2}} \sim V_b \left[\frac{N}{\sigma^2 A_b}\right]^{1/4}$$

In order to eliminate the tensile stress from this expression the proportionality $\sigma \sim T_{o2}^{-4}$, obtainable from Fig. 10, to give:

$$W_b^* \sim C_{pb} T_{o2}^{2} \left[\frac{T_{o2}}{T_{o1}} - r_c^{\lambda}\right] \left[\frac{W T_{o1}}{r_c P_{o1}}\right]^{3/4} \left[\frac{N}{R(F_{m1} + r_c^{\lambda})}\right]^{1/4}$$

The foregoing was intended to apply to metallic combustors only. Another type, intended for large marine or stationary plants and lined with refractory, requires separate treatment.

Data on current combustion equipment permit approximate evaluation of the constant of proportionality to give:

$$W_b^* = \frac{C_{pb} T_{o2}^{2}}{2000} \left[\frac{T_{o2}}{T_{o1}} - r_c^{\lambda}\right] \left[\frac{W T_{o1}}{r_c P_{o1}}\right]^{3/4} \left[\frac{N}{R(F_{m1} + r_c^{\lambda})}\right]^{1/4} \qquad (30)$$

Further, the space required for a combustion system composed of a set of identical can-type units, as opposed to the net internal or gas volume used above, may be approximated as:

$$V_b^* = \pi (1.5\, d_b)^2 L_b^*$$
$$= \frac{725\, N d^2 C_{pb}}{R} \cdot \frac{(T_{o2}/T_{o1}) - r_c^{\lambda}}{(F_{m1} + r_c^{\lambda})}$$

By making use of the flame cross-sectional area relation for proper gas velocity mentioned at the beginning of this section, this becomes:

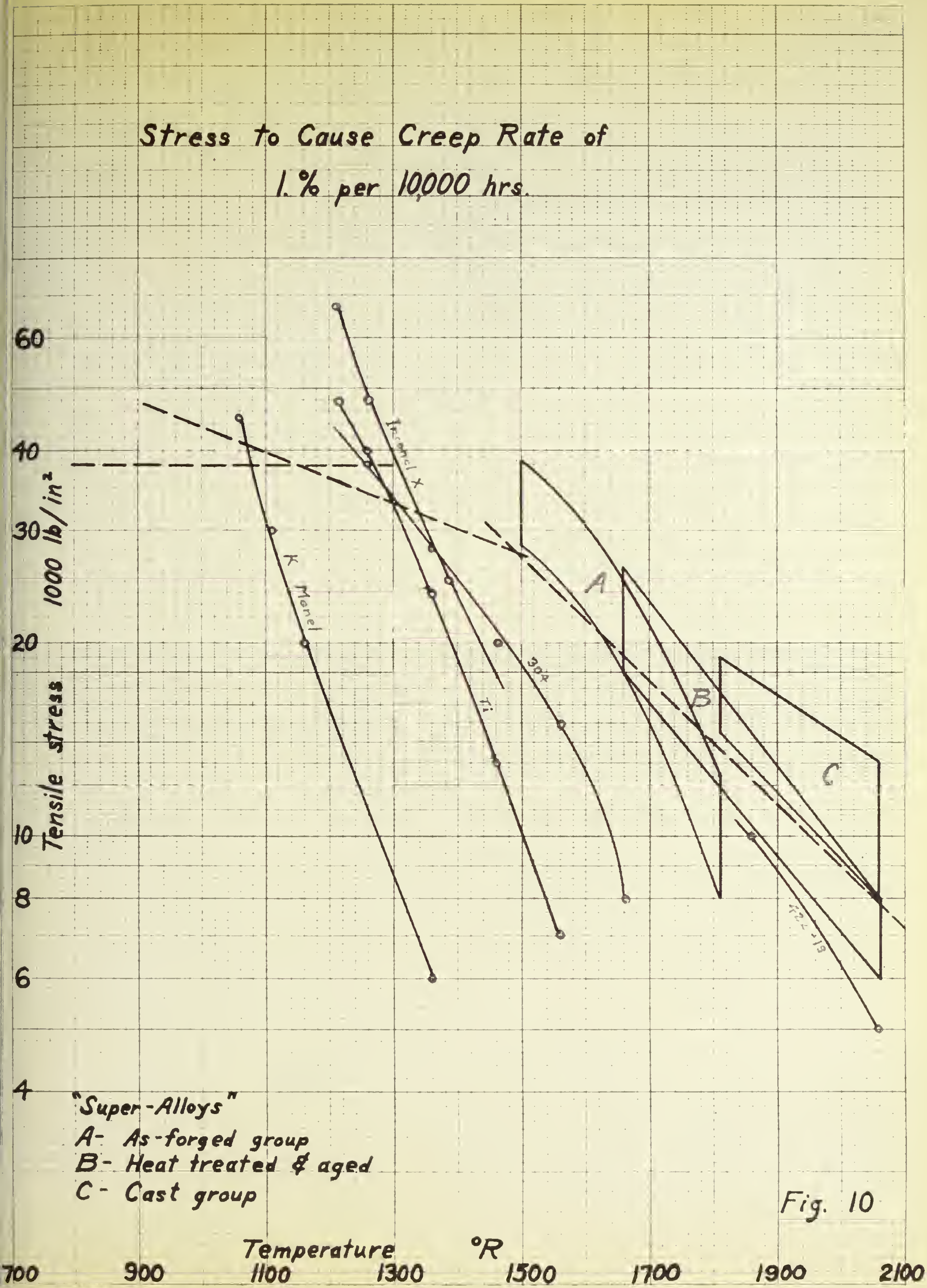

Stress to Cause Creep Rate of
1.% per 10,000 hrs.
Tensile stress 1000 lb/in²
60
40
30
20
10
8
6
4
Inconel X
K Monel
Ti
304
A
B
C
"Super-Alloys"
A- As-forged group
B- Heat treated & aged
C- Cast group
Temperature °R
700
900
1100
1300
1500
1700
1900
2100

Fig. 10

$$V_b^* = 2.31 \frac{C_{pb}T_{o1}P^*}{P_{o1}} \left[\frac{(T_{o3}/T_{o1})-r_c^{\lambda}}{r_c} \left(\frac{w}{P^*}\right)\right] \qquad (31)$$

The trend of combustor volume with pressure ratio as given by this is shown for the single temperature ratio $(T_{o3}/T_{o1}) = 3.7$ in Fig. 4.

The construction of stators and casings is considered to be governed by regard for stiffness rather more than for rupture strength. Stator blading weight is assumed to average the same as that of rotor blading operating within it. Following ref. 5 and the foregoing treatment of combustor shell weight, the stator blading and casing weights become:

$$W^* = \frac{\pi(L/d)^2 d^3 \rho_b}{\delta} \left[\frac{t\phi}{2(L/d)}\right] \qquad \text{for the blades}$$

$$W^* \sim V_r^* \left[\frac{1}{\sigma^2 D_r^2}\right]^{1/4} \sim d_r^{3/2} \left[1-(L/d)\right]^{3/2} L_r^* \qquad \text{for the casing}$$

Again the constant is approximated on the basis of current plants to give, for the compressor:

$$W_{cc}^* = \pi d_c^{3/2} \rho_b \left[\frac{t\phi(L/d)_1^2 d_c^{3/2}}{2\delta} + \left[.006\ 1+(L/d)_1\right]^{3/2} L_c^*\right] \qquad (32)$$

and for the turbine:

$$W_{tc}^* = \pi d_t^{3/2} \rho_b \left[\frac{t\phi(L/d)_4^2 d_t^{3/2}}{2\delta} + .01 \left[1+(L/d)_4\right]^{3/2} L_t^*\right] \qquad (32a)$$

both of which increase monotonically with (L/d).

Equally defensible is the approximation to casing volume as, in the case of the compressor:

$$V_{cc}^* = V_c \left[\frac{D+2b}{D}\right]^3 = V_c(1+2b/D)^3$$

but since

$$\delta = (L/d)\ (d/b) \qquad \text{and} \quad b/D = (b/d)(d/D)$$

by (9) there appears that:

$$V^{*}_{cc} \approx V_c \left[1+6b/D \right] = V_c \left[1+ \frac{(6/\delta_{mc})(L/d)_1}{1+(L/d)_1} \right] \tag{33}$$

and similarly for the turbine casing volume.

$$V^{*}_{tc} = V_t \left[1 + \frac{(6/\delta_{mt})(L/d)_4}{1+(L/d)_4} \right] \tag{33a}$$

SCROLLS, VOLUTES and DUCTS

Scrolls, volutes, ducts, piping, stacks and so on vary greatly from plant to plant for the good reason that plant location and job assignment dictate component layout to a large extent. The straight-through aircraft jet engine with axial compressor certainly has the minimum of such parts, whereas a marine propulsion or shore power generation plant has a good deal of its total weight and space so constituted. Hence no definitive mathematical treatment can be shown, but a few general rules can be given.

For the simple cycle, gas pressures up to the compressor inlet and beyond the turbine exit are of the order of one atmosphere. Within these sections of the flow path design is considered to be based on rigidity rather than bursting pressure. Hence, following the reasoning of ref 9:

$$V^* \sim w$$

$$W^* \sim w$$

From the compressor discharge to the turbine nozzle the flow path carries the same weight flow at a pressure r_c times as great and a specific volume correspondingly less. In this section design is considered to be based on bursting strength, but in the first approximation it turns out again that:

$$W^* \sim w$$

while

$$V^* \sim w/r_c$$

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$$W^* \sim W$$

while

$$V^* \sim W/r_c$$

Since it has been shown that the net power output for given cycle conditions is:

$$P^* \sim w$$

then it may be concluded that for a given type of plant -- aircraft, mobile or stationary -- the size and volume of ducting will vary nearly linearly with the mass flow, once the cycle thermodynamic conditions are fixed. Taking account of the relation between air rate and pressure ratio, the weight and size of ducting and connections may be expressed as a function of the variable (w/P^*) to fit any plant capacity.

VII. APPLICATION

In order to clarify the effect of certain variables as they individually influence the design, it is advisable to fix as many of the other variables as possible, or in other words to take a particular plant and inquire as to how the internal geometry of the components may be optimized to produce the most power for the least investment.

Choosing a simple CBTG plant (Fig. 1) for this purpose, the following points are fixed:

Compressor	Turbine
$T_{o1} = 530°R$	$T_{o3} = 1960°R$
$C_p = .24$	$C_p = .27$
$\eta_{sc} = .90$	$\eta_{sc} = .85$
$\beta_2 = 60°$	$\beta_2 = 30°$
$\theta = 40°$	$\theta = 60°$
$t = .10$	$t = .15$
$\sigma_b = 22{,}000$ psi	$\sigma_b = 24{,}000$ psi
$\sigma_g = 4000$ psi	$\sigma_g = 5000$ psi

Both machines:

$r = 5$	$\tau = .7$
$R = 53.3$ ft./°R	$\rho_b = \rho_d = 500$ lb./ft.^{3}
$\gamma = 1.3$	$P_{o1} = P_{o4} = 2116$ lb./ft.^{2}

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Choosing a simple CGTG plant (Fig. 1) for this purpose, the following points are fixed:

Compressor	Turbine
$T_{01} = 530°R$	$T_{03} = 1960°R$
$C_p = .24$	$C_p = .27$
$\eta_{sc} = .90$	$\eta_{sc} = .88$
$\beta_a = 60°$	$\beta_a = 20°$
$\theta = 40°$	$\theta = 60°$
$t = .10$	$t = .15$
$\sigma_b = 22,000$ psi	$\sigma_b = 24,000$ psi
$\sigma_s = 4000$ psi	$\sigma_s = 5000$ psi

Both machines:

$r = 5$	$\gamma = .7$
$R = 53.3$ ft./°R	$p_b = p_a = 500$ lb./ft.^{2}
$Y = 1.3$	$p_{01} = p_{04} = 2116$ lb./ft.^{2}

From these it follows that (Fig. 11 et seq):

$$F_{m_i} = .1 \qquad F_{m_s} = .03$$

$$F_{bc} = 2.85 \qquad F_{bt} = 6.58$$

$$\lambda = .318 \qquad \mu = .211$$

$$r_c^{\lambda} = 1.67 \qquad r_t^{-\mu} = .711$$

$$r_c^{1-\lambda} = 3.0 \qquad r_t^{1-\mu} = 3.56$$

$$F_c = .5 \qquad F_t = .439$$

$$\Sigma_c = 9050. \qquad \Sigma_t = 9870.$$

$$w/P^* = .0105 \text{ lb/HP sec}$$

$$\phi_c = .96 \qquad \phi_r = 1.08$$

Having decided to investigate the effect of the flow coefficient (C_x/U), blade length ratio (L/d) and mass flow (w), all other parameters are expressed in terms of these three by using the information above and the equations set forth previously. That these three variables are independent may be seen from the continuity relation and (3). Under the original simplifying assumptions and the above set conditions none of these three variables affect the thermodynamics of the cycle, however, and so the air rate (w/P^*) remains unchanged. The effect of scaling up the power rating of the plant, then, for any fixed set of values such as the above may be taken directly as the effect of increasing the mass flow rate.

Equations here are numbered to correspond to their counterparts in the preceding general development. They are:

From these it follows that (Fig. 11 et seq):

$$P_{ii} = 1. \qquad P_{ms} = .03$$

$$P_{bo} = 3.38 \qquad P_{nt} = 6.50$$

$$\lambda = .318 \qquad \mu = .211$$

$$r_c^{\lambda} = 1.67 \qquad r^{-\mu} = .711$$

$$r_c^{1-\lambda} = 3.0 \qquad r^{1-\frac{1}{\mu}} = 3.55$$

$$P_c = .5 \qquad P_t = .453$$

$$\Sigma_c = 9060. \qquad \Sigma_t = 3870.$$

$$w/P^* = .0100 \text{ lb/HP sec}$$

$$\eta_c = .86 \qquad \eta_r = 1.06$$

Having decided to investigate the effect of the flow coefficient (C_x/U), blade length ratio (L/d) and mass flow (w), all other parameters are expressed in terms of these three by using the information above and the equations set forth previously. That these three variables are independent may be seen from the continuity relation and (3). Under the original simplifying assumptions and the above set conditions none of these three variables affect the thermodynamics of the cycle, however, and so the air rate (w/P*) remains unchanged. The effect of scaling up the power rating of the plant, then, for any fixed set of values such as the above may be taken directly as the effect of increasing the mass flow rate.

Equations here are numbered to correspond to their counterparts in the preceding general development. They are:

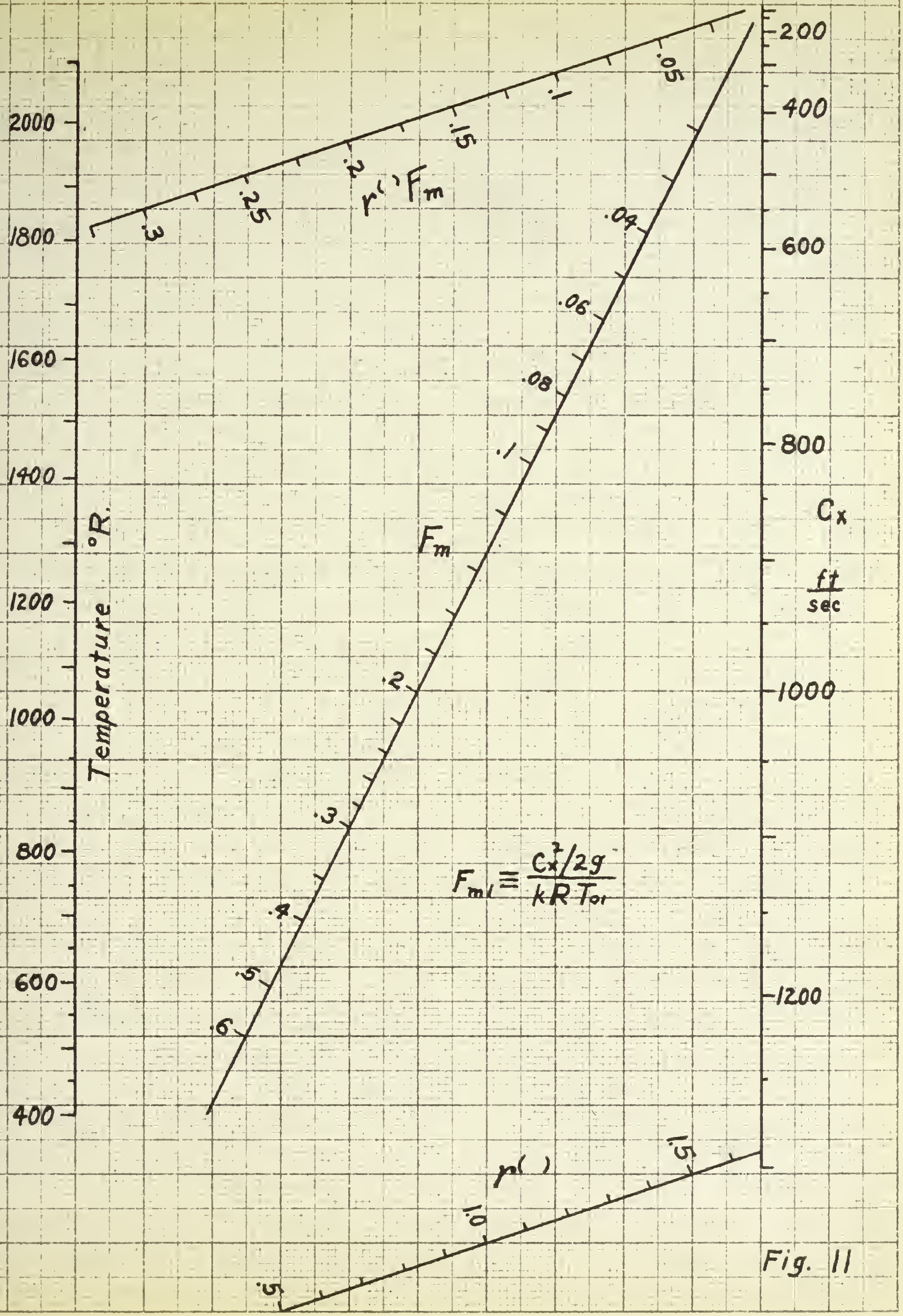

2000
1800
1600
1400
1200
1000
800
600
400
Temperature °R.
.3
.25
.2
.15
.1
.05
r( ) Fm
.04
.06
.08
.1
Fm
.2
.3
.4
.5
.6
$F_{m1} \equiv \frac{C_x^2/2g}{kRT_{01}}$
.5
1.0
1.5
p( )
200
400
600
800
1000
1200
Cx
ft/sec

Fig. 11

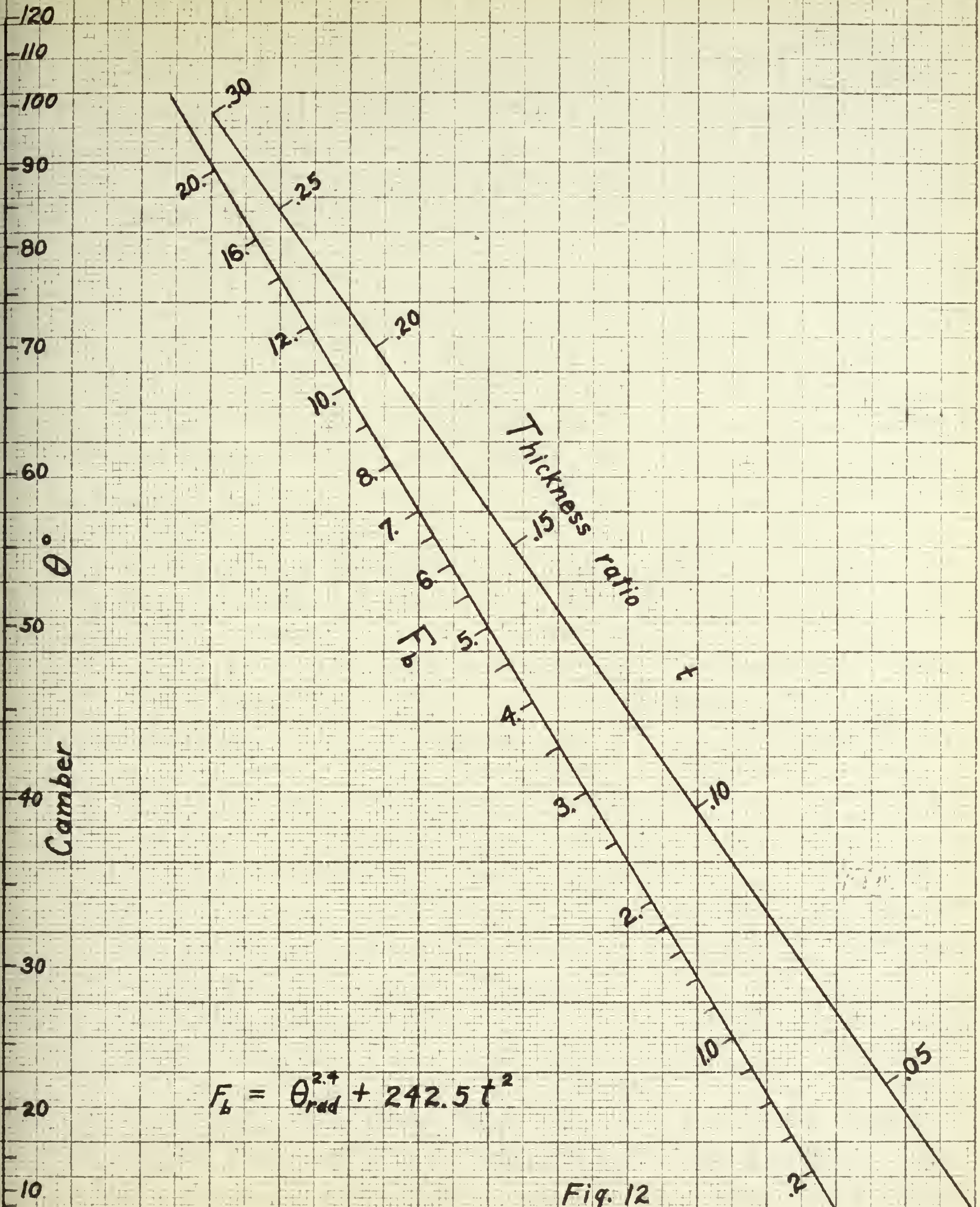
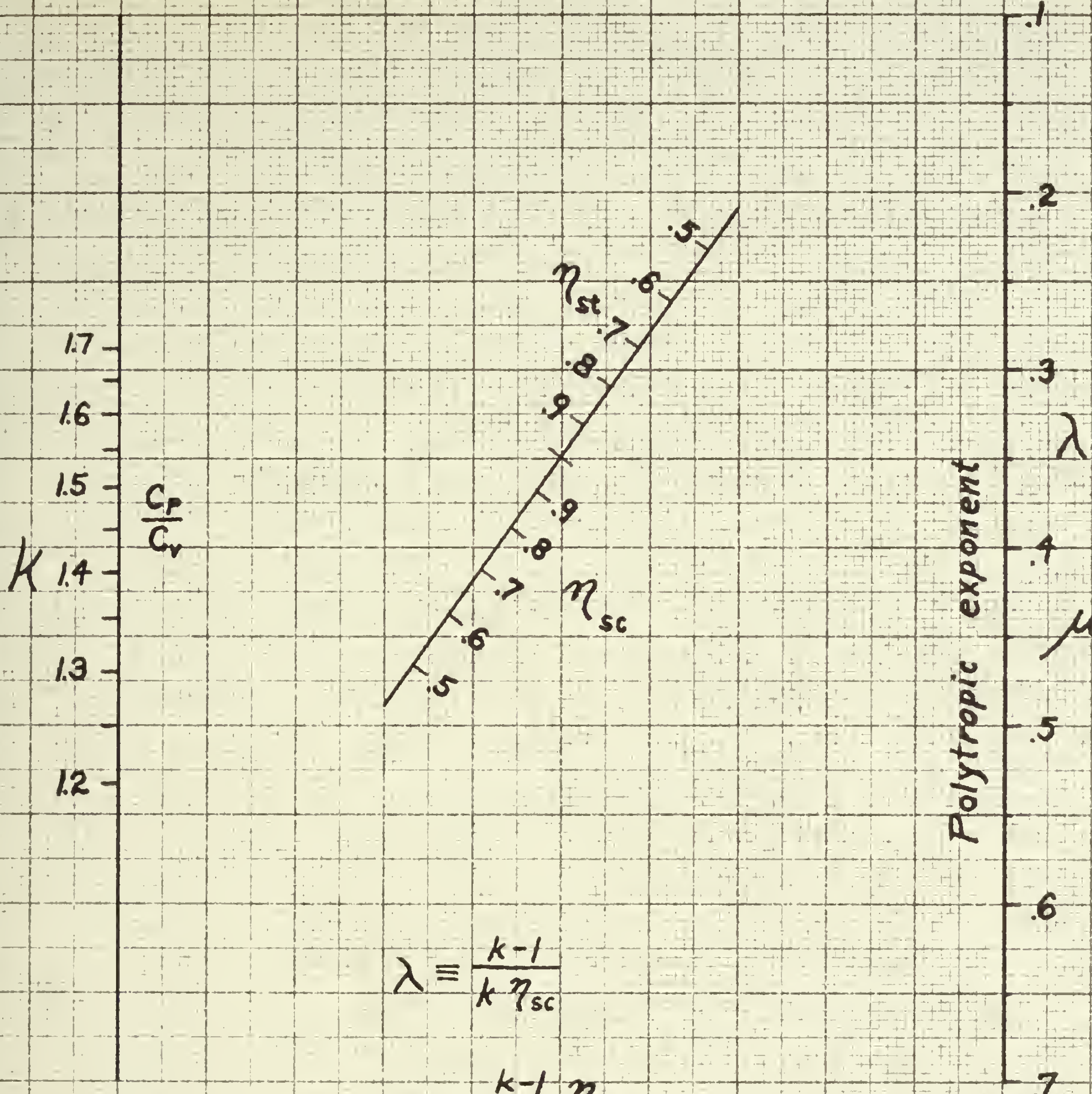

120
110
100
90
80
70
60
50
40
30
20
10
Camber
$\theta°$
.30
.25
.20
.15
.10
.05
Thickness ratio
t
20.
16.
12.
10.
8.
7.
6
5.
4.
3.
2.
1.0
.2
F_b
$F_b = \theta_{rad}^{2.4} + 242.5\,t^2$

Fig. 12



$$\lambda \equiv \frac{k-1}{k\,\eta_{sc}}$$

$$\mu \equiv \frac{k-1}{k}\,\eta_{st}$$

Fig. 13

$$d_c = .1106 \frac{w^{1/2}}{(C_x/U)_1^{1/2}\ (L/d)_1^{1/4}} \qquad (8)$$

$$d_t = .171 \frac{w^{1/2}}{(C_x/U_4)^{1/2}\ (L/d)_4^{1/4}} \qquad (8a)$$

$$G = .674 \left[\frac{(C_x/U_4)}{(C_x/U)_1}\right]^{1/2} \left[\frac{(L/d)_1}{(L/d)_4}\right]^{1/4} \qquad (15)$$

$$\delta_{mc} = 3.15 \left[\frac{(L/d)_1}{(C_x/U)_1}\right]^{1/2} \qquad (25)$$

$$\delta_{mt} = 4.84 \left[\frac{(L/d)_4}{(C_x/U)_4}\right]^{1/2} \qquad (25a)$$

$$\delta_{mc}/\delta_{mt} = 1.43\ G^2$$

$$\left[\frac{W}{U^2/2g}\right]_c = 1.024\ (C_x/U)_1 \qquad (18)$$

$$\left[\frac{W}{U^2/2g}\right]_t = 3.46\ (C_x/U)_4 \qquad (18)$$

$$V^*_{cc} = .0282 \frac{w^{3/2}\ (L/d)_1^{3/4}}{(C_x/U)_1^{3/2}} \left[1 + \frac{1.9(C_x/U)_1(L/d)_1^{1/2}}{1 + (L/d)_1}\right]\left[1+(L/D)_1\right]^2 \qquad (33)$$

$$V^*_{tc} = .0252 \frac{w^{3/2}\ (L/d)_4^{3/4}}{(C_x/U)_4^{3/2}} \left[1 + \frac{1.24(C_x/U)_4(L/d)_4^{1/2}}{1 + (L/d)_4}\right]\left[1+(L/d)_1\right]^2 \qquad (33a)$$

These last two may also be written:

$$V^*_{cc} = .00504(w\delta_{mc})^{3/2}\left[1 + \frac{(6/\delta_{mc})(L/d)_1}{1+(L/d)_1}\right]\left[1+(L/d)_1\right]^2$$

$$V^*_{tc} = .00237\,(w\,\delta_{mt})^{3/2}\left[1+\frac{(6/\delta_{mt})(L/d)_4}{1+(L/d)_4}\right]\left[1+(L/d)_4\right]^2$$

$$W^*_c = .0433\,\frac{w^{3/2}}{(C_x/U)_1^{5/2}}\left[(L/d)_1^{1/4}\,(W^*_s/d_c^{\,3})\right] \qquad (28a)$$

$$W^*_t = .057\,\frac{w^{3/2}}{(C_x/U)_4^{3/2}}\left[(L/d)_1^{1/4}\,(W^*_s/d_c^{\,3})\right] \qquad (28b)$$

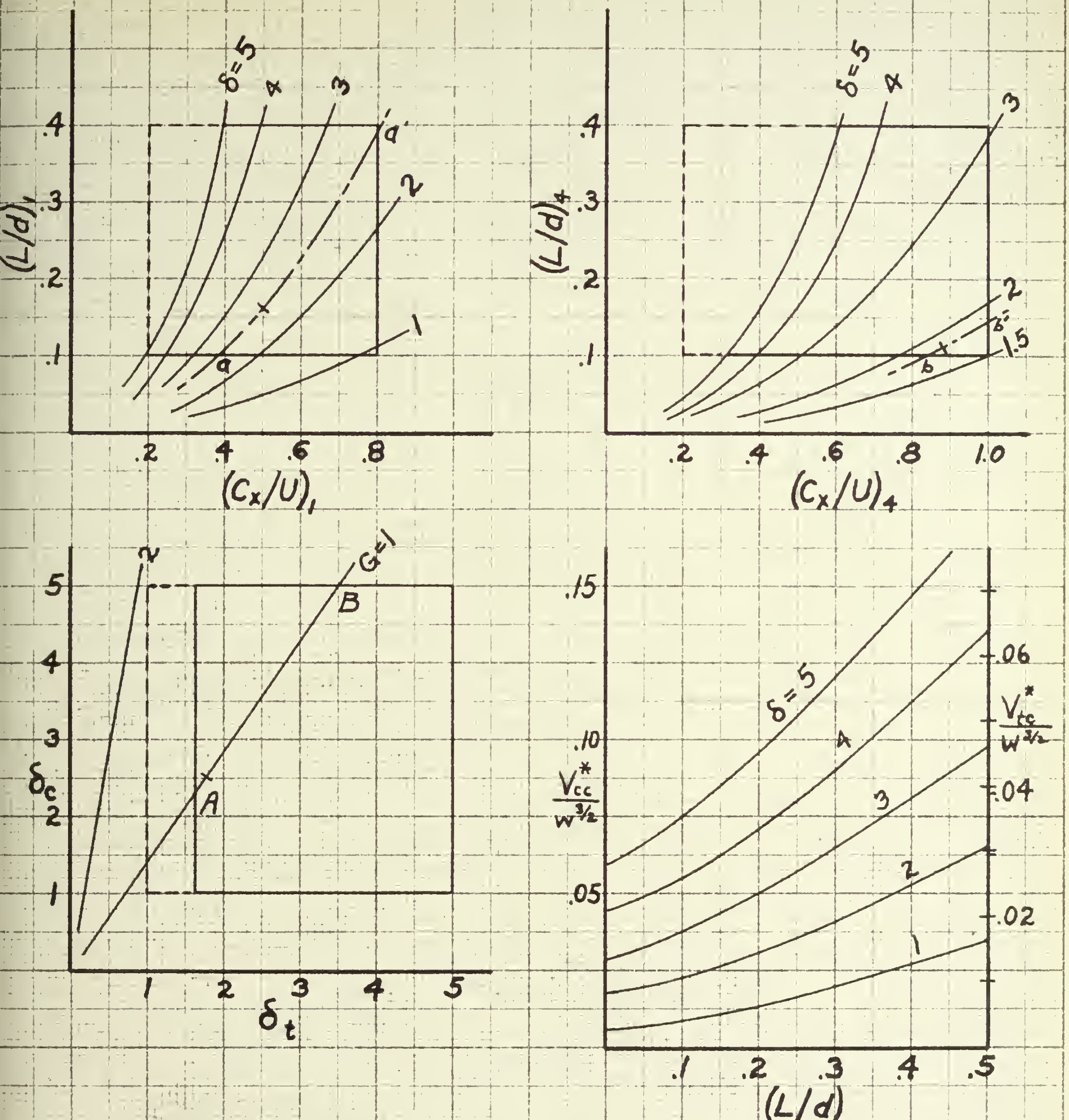
$$W^*_{co} = \frac{1.02\,w^{5/4}\,(L/d)_1^{7/8}}{(C_x/U)_1^{5/4}}\left[.0318\,\frac{(C_x/U)_1^{3/4}\,w^{1/4}}{(L/d)_1^{1/8}} + \left[1+(L/d)_1\right]^{3/2}\right] \qquad (32)$$

$$W^*_{tc} = \frac{1.23\,w^{5/4}\,(L/d)_4^{7/8}}{(C_x/U)_4^{5/4}}\left[.108\,\frac{(C_x/U)_4^{3/4}\,w^{1/4}}{(L/d)_1^{1/8}} + \left[1+(L/d)_4\right]^{3/2}\right] \qquad (32a)$$

$$V^*_b = .0611\,w$$

$$W^*_b = 53.8\,w^{3/4}$$

From (15), (25) and (25a) above the curves of Fig. 14 have been drawn. As brought out earlier the aspect ratio has been found in practice to be restricted, by loss considerations, to upper and lower limits of about five and one, respectively. Likewise, the blade length ratio (L/d) is restricted at the upper limit by blade stresses and the mechanics of construction, while restricted at the lower limit by



Choosing the Design Point

Pressure Ratio 5

Fig. 14

the fact that large physical dimensions become necessary to pass a required mass rate of flow.

Moreover the flow coefficient (C_x/U) is restricted at the lower limit by this same inability of a machine to pass sufficient mass flow, and at its upper limit is restricted by the permissible Mach No. incident to the compressor rotor blades. For example this limit may be so set that the relative velocity

$$w_1 \leq .8\sqrt{KgRT_o}$$

Then since by geometry

$$\frac{w_1}{U} = \frac{(C_x/U)}{\sin \beta_1}$$

there is

$$(C_x/U) \leq .5656 \sin \beta_1 \sqrt{(2g/U^2)KRT_o}$$

Obtaining from (3) for the example in hand

$$(U^2/2g) = 2262/(L/d)_1$$

then, for the compressor inlet:

$$(C_x/U)_1 \leq 1.585 \,(L/d)_1^{1/2}$$

or

$$\frac{(C_x/U)_1}{(L/d)_1^{1/2}} \leq 1.585$$

Hence, by (25):

$$\delta_{mo} \geqq 1.98$$

and by a similar procedure for the turbine:

$$\delta_{mt} \geqslant 1.16$$

Neither of these, it so happens, offers any additional restriction here.

With all the foregoing in mind the dotted rectangles of Fig. 14 were constructed to the (L/d) and (C_x/U) limits, the portion of such rectangles within acceptable limits of then being taken as the area of possible designs. It should be emphasized that the demarcation of these areas is carried out to show that such limits exist, rather than to pretend to lay down their numerical values.

Possible gear ratios for the plant constants chosen are indicated. Direct drive, G = 1, appears to be appropriate here since that line is the only one intersecting the rectangle properly. If direct drive is chosen, the gear may be dispensed with and the design latitude remaining is represented by the straight line segment A-B. Re-examination of equations (33) in their latter form above shows a design point near A to be preferred since lower values of aspect ratio assist in reducing turbine and compressor volume. If the point is so chosen that:

$$\delta_{mt} = 2.5$$

$$\delta_{mt} = 1.7$$

then the choice still remains open for the compressor along the arc a-a' and for the turbine along the arc b-b'.

and by a similar procedure for the turbine:

$$\delta_{mt} \cong 1.13$$

Neither of these, it so happens, offers any additional restriction here.

With all the foregoing in mind the dotted rectangles of Fig. 14 were constructed to the (L/D) and (C_W/T) limits, the portion of each rectangle within acceptable limits of them being taken as the area of possible designs. It should be emphasized that the demarcation of these areas is carried out to show that such limits exist, rather than to pretend to lay down their numerical values.

Possible gear ratios for the plant constants chosen are indicated. Direct drive, G = 1, appears to be appropriate here since that line is the only one intersecting the rectangle properly. If direct drive is chosen, the gear may be dispensed with and the design latitude remaining is represented by the straight line segment A-B. Re-examination of equations (55) in their latter form above shows a design point near A to be preferred since lower values of aspect ratio assist in reducing turbine and compressor volume. If the point is so chosen that:

$$\delta_{mc} = 3.5$$

$$\delta_{mt} = 1.7$$

then the choice still remains open for the compressor along the arc a-a' and for the turbine along the arc b-b'.

Again (33) shows the lower end of each range to be preferred in order to gain a lower value of (L/d). Hence the design may be fixed at:

$$(C_x/U)_1 = .5 \qquad (C_x/U)_4 = .88$$

$$(L/d)_1 = .16 \qquad (L/d)_4 = .11$$

and with these in hand the size and weight of the plant may be calculated from the foregoing equations, making use of the air rate, .0105 lb/HP sec., to express the results in terms of power output rather than mass flow rate.

This leads to, for the pressure ratio of five:

$$V^* = V^*_{cc} + V^*_{tc} + V^*_b$$
$$= \left(.49\ P^{*1/2} + 6.42\right)10^{-4}\ P^* \qquad \text{ft.}^3$$

and

$$W^* = W^*_c + W^*_{cc} + W^*_t + W^*_{tc} + W^*_b$$
$$= \left(47.22\ P^{*1/2} + 28.8\ P^{*1/4} + 17{,}700/P^{*1/4}\right)10^{-4} P^* \qquad \text{lbs.}$$

In order to discover the influence of pressure ratio changes on the plant size and weight, the detailed calculations may be repeated for ratios of seven and ten, with other data remaining the same. Following that, by the same criteria as before, there are obtained rectangles which overlie as shown in Fig. 15. When design values of δ, (L/d) and (C_x/U) have been chosen the values for all three pressure ratios may be tabulated as:

Again (33) shows the lower end of each range to be preferred in order to gain a lower value of (L/d). Hence the design may be fixed at:

$$(C_x/U)_1 = .6 \qquad (C_x/U)_4 = .39$$

$$(L/d)_1 = .16 \qquad (L/d)_4 = .11$$

and with these in hand the size and weight of the plant may be calculated from the foregoing equations, making use of the air rate, .0105 lb/HP sec., to express the results in terms of power output rather than mass flow rate.

This leads to, for the pressure ratio of five:

$$V^* = V^*_{cc} + V^*_{tc} + V^*_{b}$$

$$= \left(.43\, p^{*1/2} + 5.42\right) 10^{-4}\, p^* \quad ft.^3$$

and

$$W^* = W^*_c + W^*_{cc} + W^*_t + W^*_{hc} + W^*_b$$

$$= \left(47.22\, p^{*1/2} + 23.9\, p^{*1/4} + 17{,}700/p^{*1/4}\right) 10$$

In order to discover the influence of pressure ratio changes on the plant size and weight, the detailed calculations may be repeated for ratios of seven and ten, with other data remaining the same. Following that, by the same criteria as before, there are obtained rectangles which overlie as shown in Fig. 15. When design values of δ, (L/d) and (C_x/U) have been chosen the values for all three pressure ratios may be tabulated as:

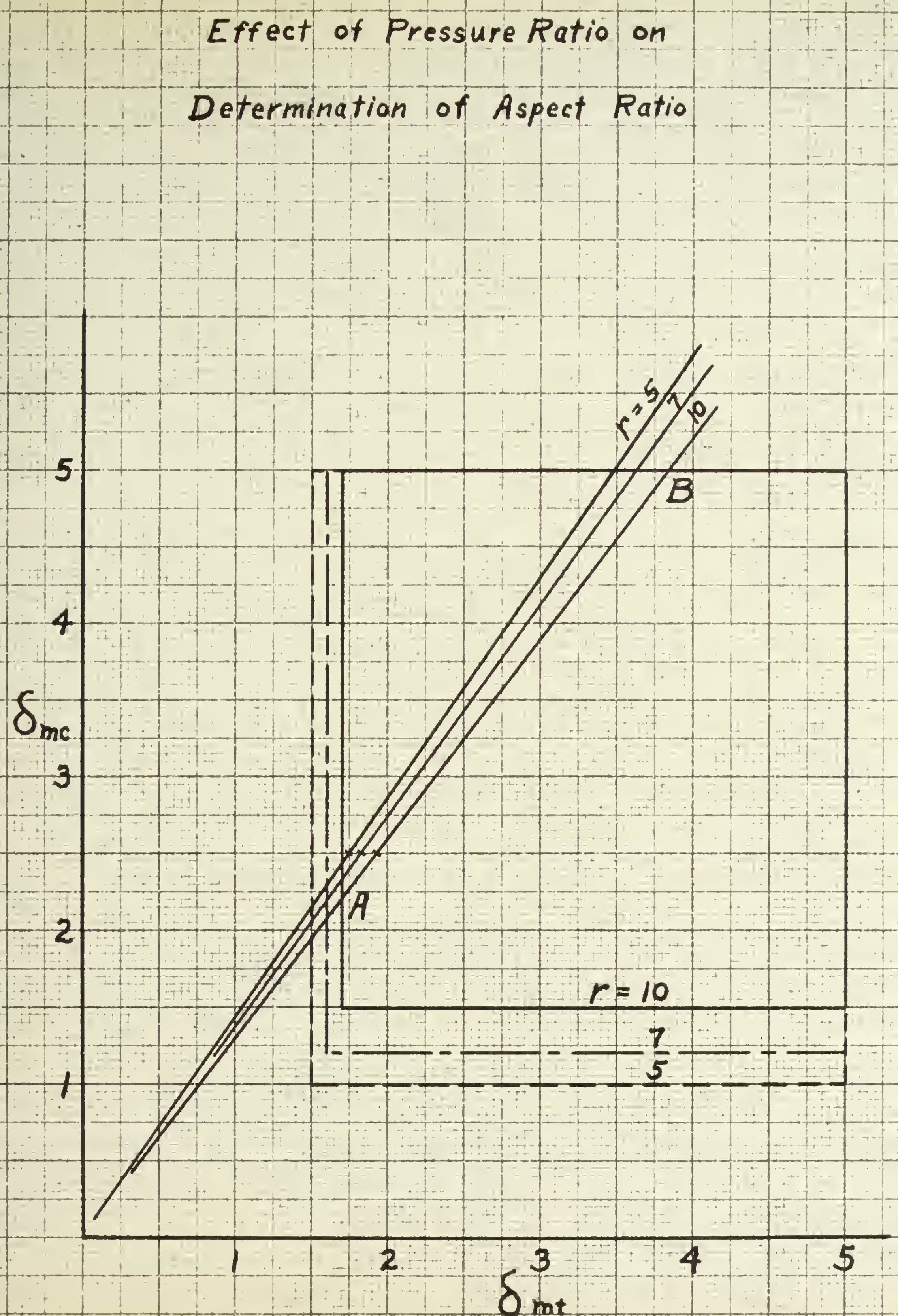
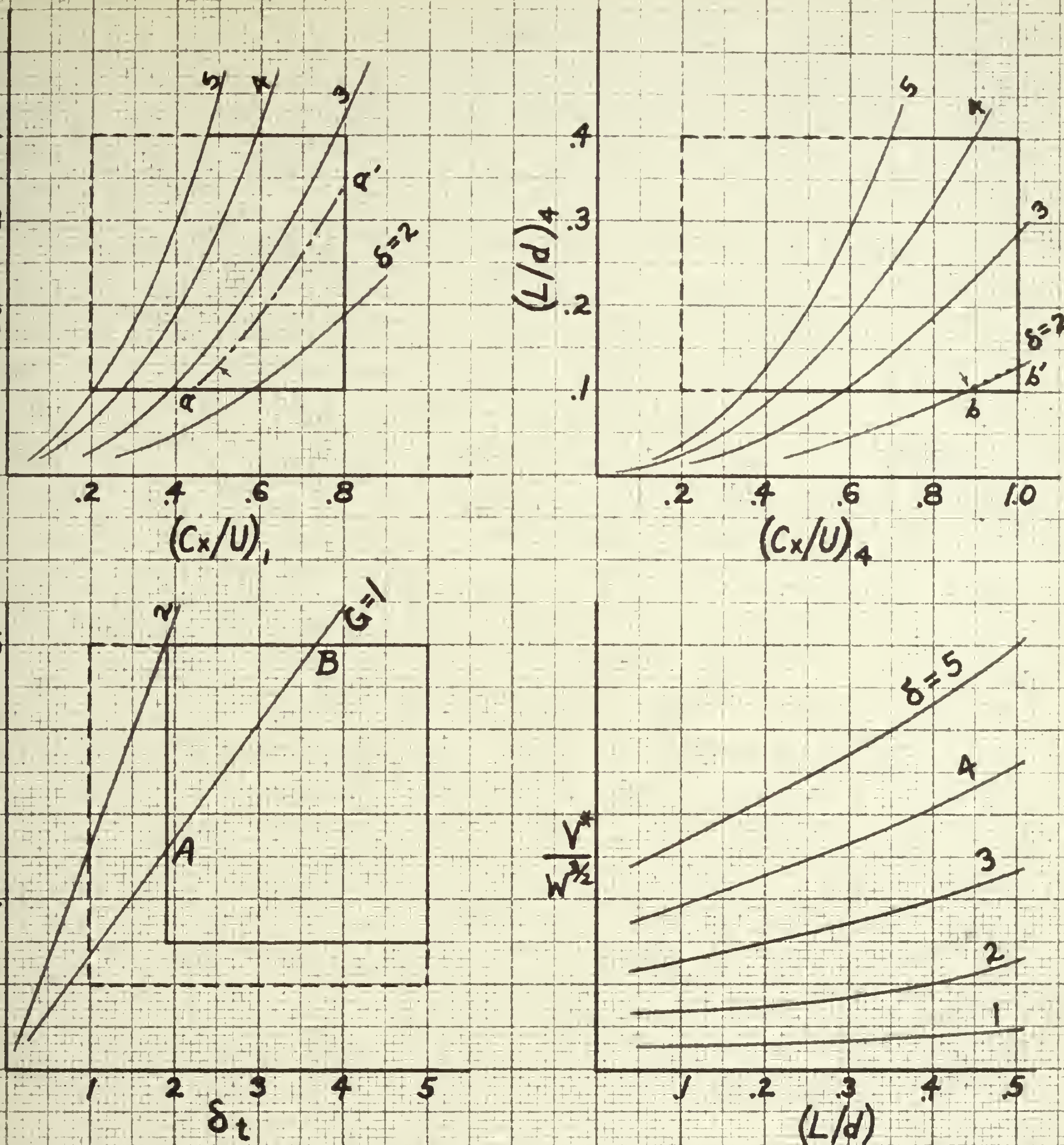

Effect of Pressure Ratio on
Determination of Aspect Ratio
r = 5
7
10
B
A
r = 10
7
5
δmc
δmt
1
2
3
4
5

Fig. 15



Choosing the Design Point

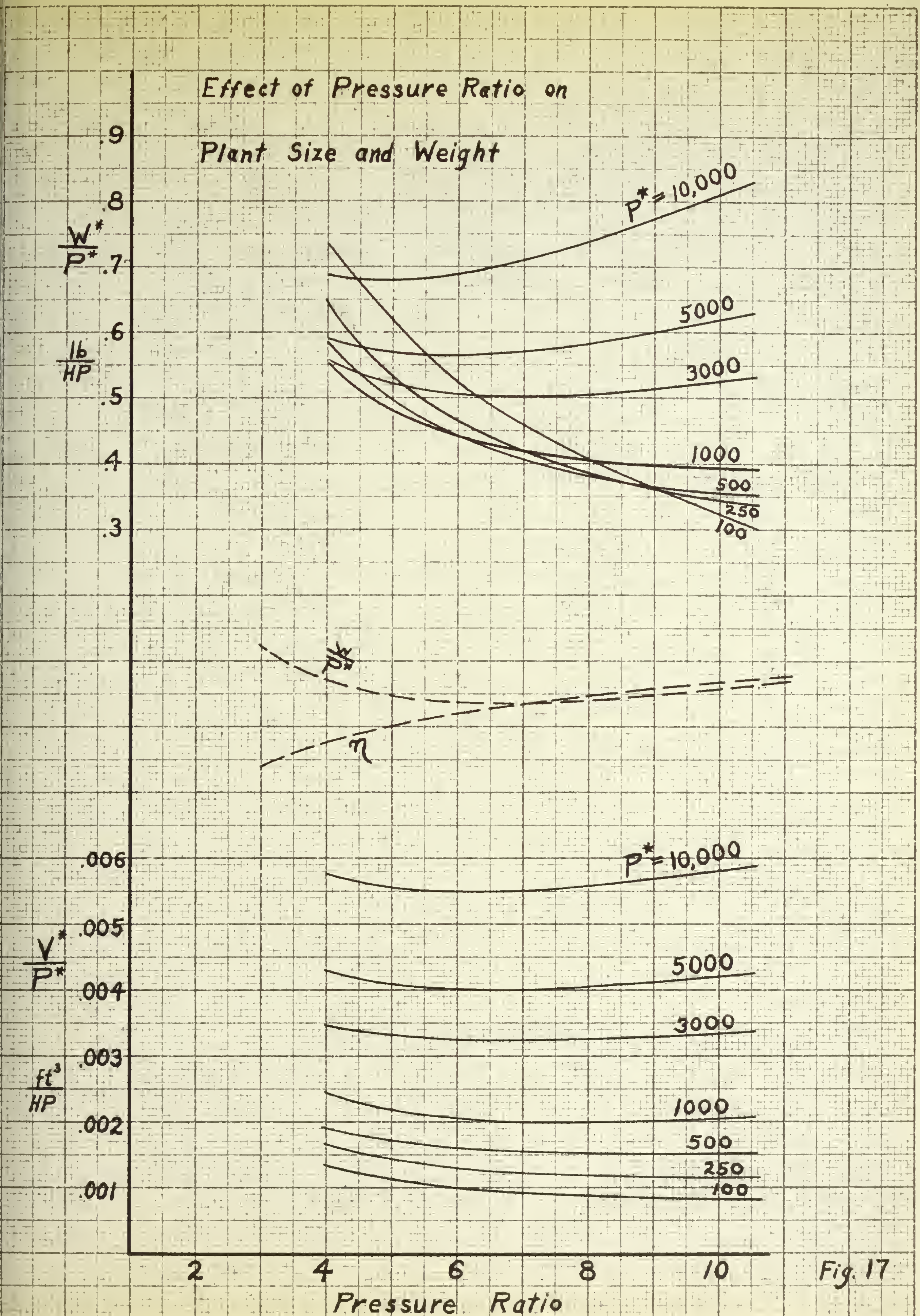
Pressure Ratio 7

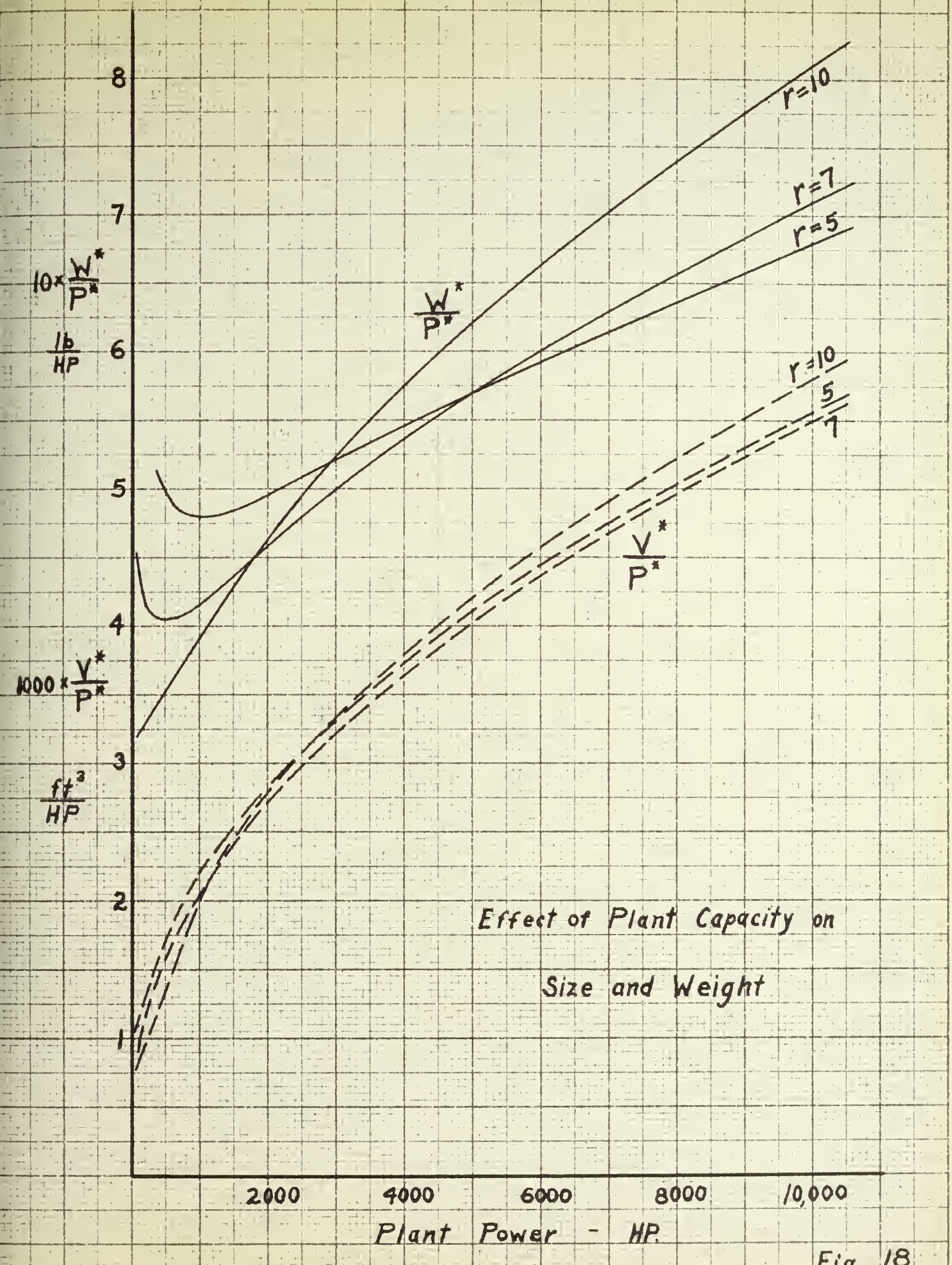
Fig. 16

r	5	7	10
δ_{mc}	2.5	2.5	2.5
$(C_x/U)_1$	.5	.5	.5
$(L/d)_1$	.16	.145	.13
δ_{mt}	1.7	1.85	1.95
$(C_x/U)_4$	.88	.88	.87
$(L/d)_4$	.11	.11	.11

It may be concluded that in the range covered by CBT plants a pressure ratio variation effects little change in the choice of these parameters in turbine and compressor. On the other hand equations (11) et seq. show that the stress parameter Σ, the blade angle and camber β and θ, and the degree of aerodynamic loading under which the blades operate are the three major factors which in the end direct the selections of (L/d) and (C_x/U). Component stage efficiencies bear on the thermal efficiency and the air rate, and via the latter affect the selection as well.

It may be concluded from the curves (W*/P*) of Fig. 17 that, on the basis of minimum plant weight being a desirable factor, it is advantageous at certain pressure ratios to divide a load between two or more identical plants operating in parallel at a net saving in weight. The same possibility with respect to reducing total plant space requirements is suggested by curves (V*/P) of the same figure. Mitigating against this apparent opportunity to save weight and space,


Effect of Pressure Ratio on
Plant Size and Weight
.9
.8
W*/P*
.7
.6
lb/HP
.5
.4
.3
P* = 10,000
5000
3000
1000
500
250
100
W*/P*
η
.006
.005
V*/P*
.004
.003
ft³/HP
.002
.001
P* = 10,000
5000
3000
1000
500
250
100
2
4
6
8
10
Pressure Ratio
Fig. 17



Effect of Plant Capacity on Size and Weight

Fig. 18

and in all likelihood actually reversing the trend in an actual installation, is the necessity for duplication of controls, accessories, instrumentation and servicing when sets of smaller unit size are used to fulfill a job requirement.

A quantitative analysis of these two opposing effects is suggested as a suitable subject of further investigation.

VIII. APPENDIX

Symbolism

A	-	area, annular unless otherwise noted	$ft.^2$
b	-	blade width, axially	ft.
C	-	absolute velocity	ft/sec.
C_p	-	specific heat at constant pressure	BTU/lb,°F.
D	-	tip diameter, over longest blade	ft.
d	-	pitch diameter, at mid-blade	ft.
F	-	general function; also a general force	
g	-	gravitational constant, 32.2	ft/sec^2
h	-	enthalpy	BTU/lb
I	-	moment of inertia	$ft.^4$
J	-	mechanical equivalent of heat, 778	ft.lb/BTU
k	-	ratio of specific heats, C_p/C_v	
L	-	blade length, radially	ft.
L*	-	component length	ft.
M	-	Mach No.	
m	-	bending moment	lb.ft
N	-	number of like stages or units	
P	-	pressure	$lb/ft.^2$
P*	-	power, horsepower unless otherwise shown	
R	-	gas constant, for air 53.3	ft/°F.
R*	-	reheat factor	
r	-	pressure ratio in a component, greater than unity	
S	-	blade spacing, circumferentially, on pitch circle	ft.
∅	-	solidity, b/S	

T -	temperature	°R
t -	thickness ratio, vs., chord or diameter	
U -	circumferential velocity on pitch diameter	ft/sec.
V -	net or working volume	$ft.^3$
V* -	component volume, overall	$ft.^3$
W -	work	ft.lb/lb.
W* -	component weight	lb.
w -	mass flow rate; also relative velocity	lb/sec;ft/sec.
Y -	cascade clearance factor	
y -	extreme fiber distance from neutral axis	ft.
Z -	number of blades in a row	
α -	angle of absolute velocity with plane of cascade	
β -	angle of relative velocity with plane of cascade	
Δ -	a special function of (L/d) and δ	
δ -	blade aspect ratio, L/b	
ε -	angle of fluid deflection	
η -	efficiency, output/input	
θ -	blade camber angle	
λ -	the exponent $(k-1)/k\,\eta_{sc}$	
μ -	the exponent $(k-1)\,\eta_{st}/k$	
ν -	the exponent $(k-1)/k$	
ρ -	density	$lb/ft.^3$
Σ -	stress parameter, $\sigma/\gamma\rho_b$	ft.
σ -	tensile stress	$lb/ft.^2$
τ -	taper factor, for stress reduction in rotating blade	
ψ -	aerodynamic load coefficient	

ω - angular velocity rad/sec.

Subscripts

b - burner or combustor; also blade

c - compressor; also casing or stator

g - gas-bending

j - jet

m - mean; also pertaining to Mach No.

n - net

o - stagnation state

p - constant pressure

s - stage; also static

t - turbine

x - axial direction

Station identification

1 - compressor rotor entrance

2 - compressor exit; combustor entrance

3 - combustor exit; turbine inlet

4 - turbine exit

5 - jet discharge

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